3D TEM-IP inversion workflow for galvanic source TEM data

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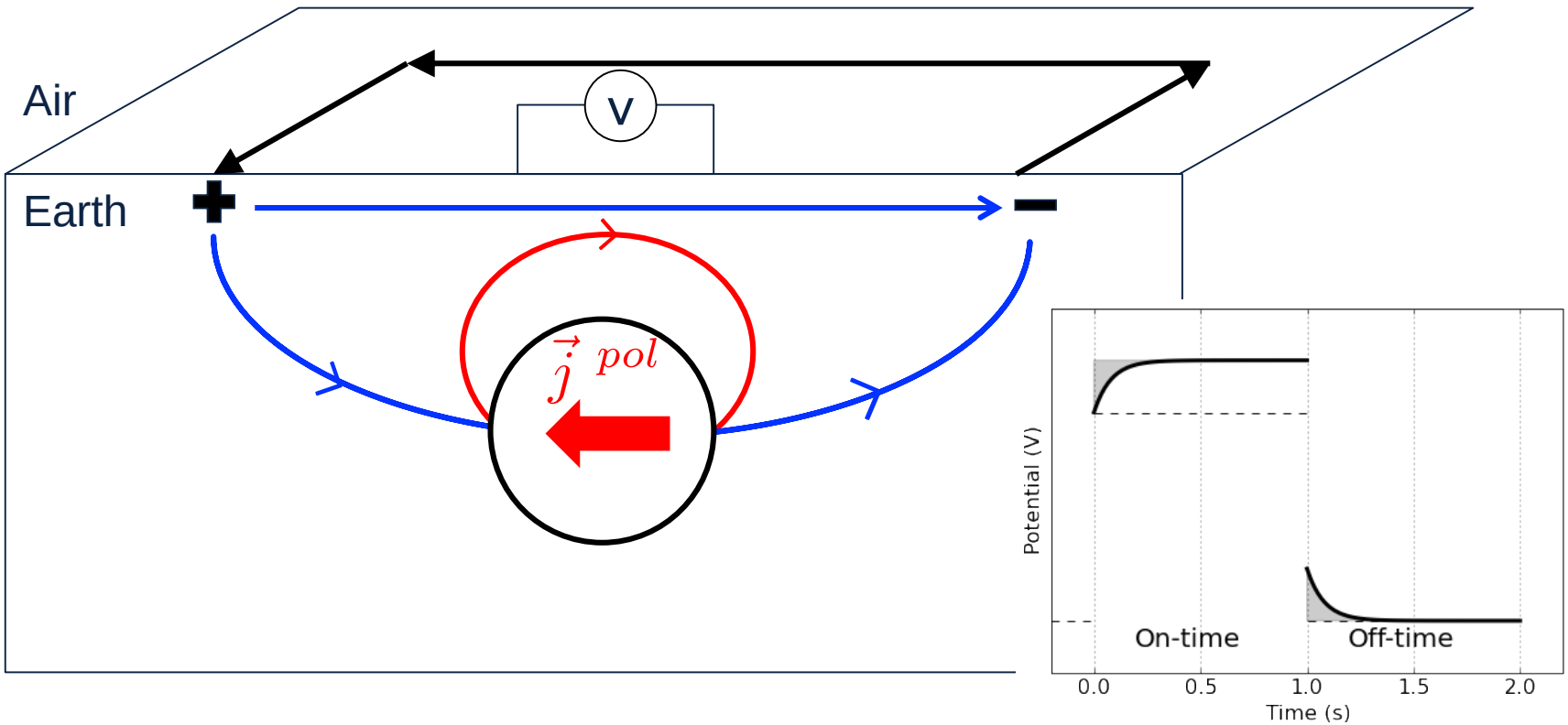
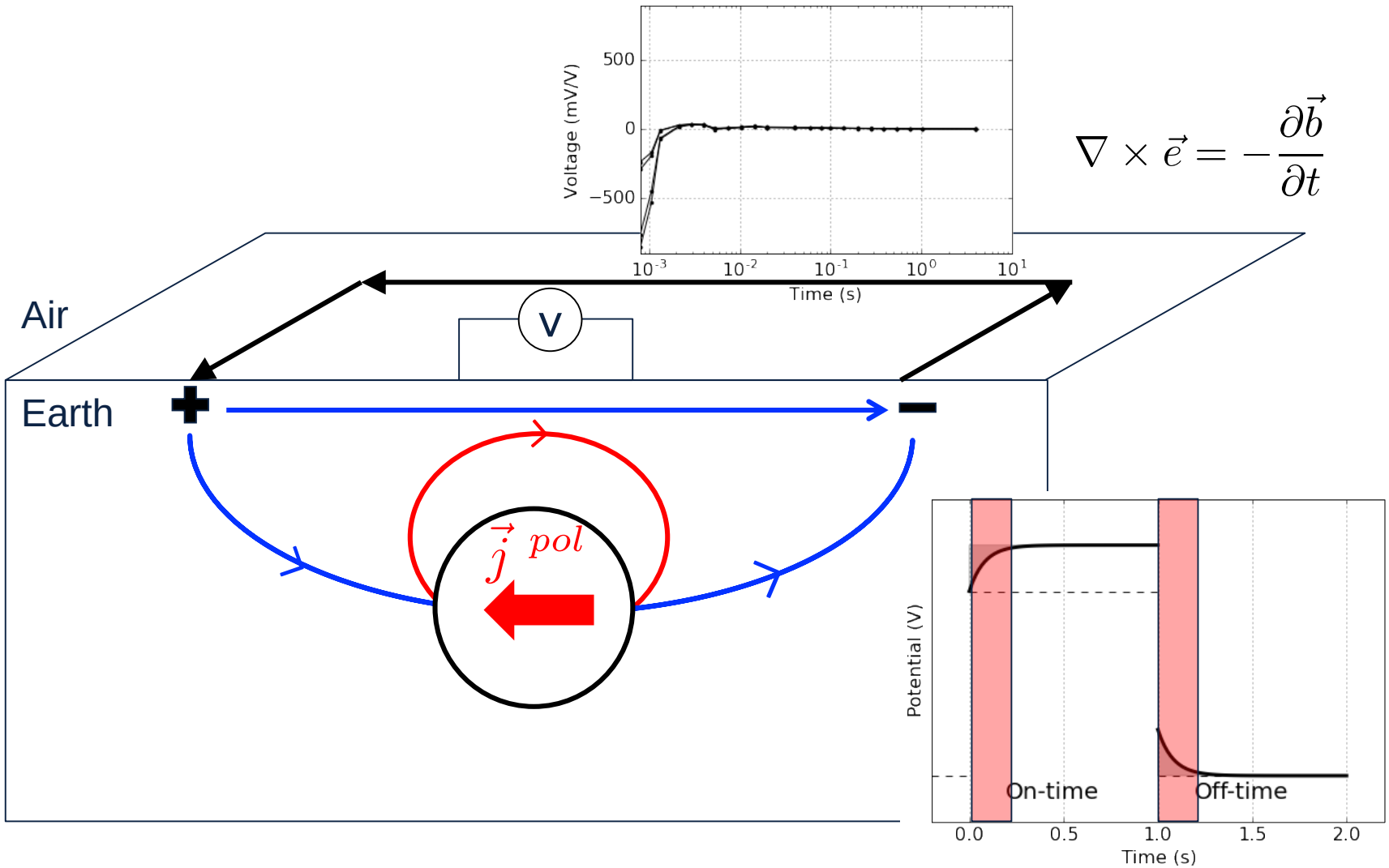
Conceptual model for IP response



EM coupling?



$$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$$

Questions?

- Measured voltage includes both EM and IP effects
  - How are we recovering conductivity and chargeability?
  - Are we okay with EM-contamination when recovering chargeability?

Simulation of TEM data

- Maxwell's equations:

Frequency domain

$$\vec{\nabla} \times \vec{E} = -\imath\omega\vec{B}$$

$$\vec{\nabla} \times \mu^{-1}\vec{B} - \vec{J} = \vec{J}_s$$

Time domain

$$\vec{\nabla} \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$$

$$\vec{\nabla} \times \mu^{-1}\vec{b} - \vec{j} = \vec{j}_s$$

Ohm's law in **frequency** domain

$$\vec{J} = \sigma\vec{E}$$

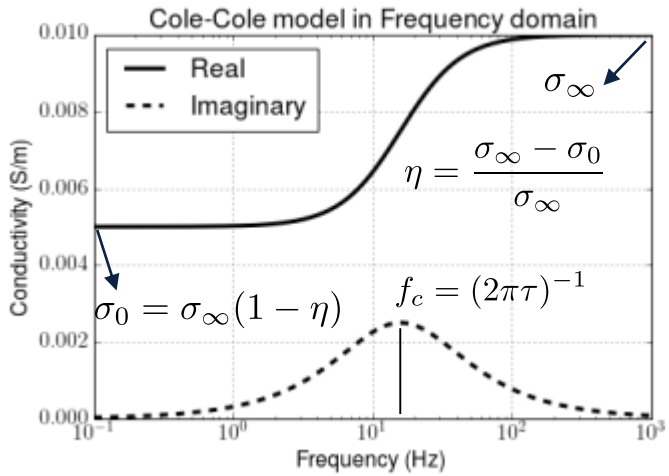
Ohm's law in **time** domain

$$\vec{j} = \sigma\vec{e}$$

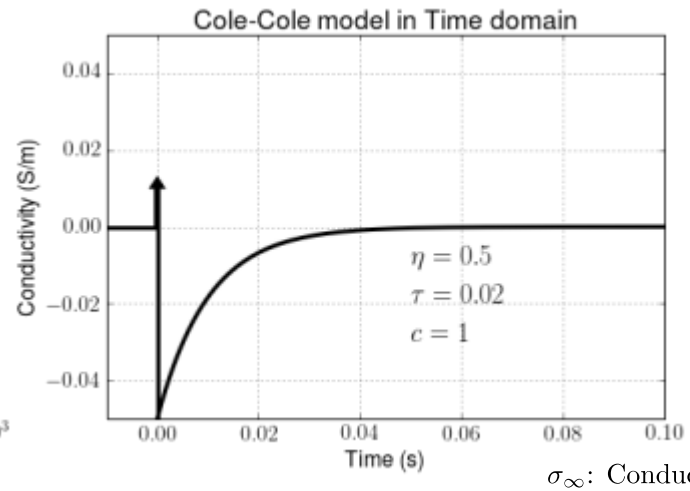
Complex conductivity

- Cole-Cole model (Pelton et al., 1978)

Frequency domain



Time domain



σ_∞: Conductivity at infinite frequency
σ_0: Conductivity at zero frequency
η: Chargeability
τ: Time constant (s)
c: Frequency dependency

Inverse Fourier transform

$$\sigma(\omega) = \sigma_\infty + \sigma_\infty \frac{\eta}{1 + (1-\eta)(\imath\omega\tau)^c} \longrightarrow \mathcal{F}^{-1}[\sigma(\omega)] = \sigma(t)$$

Simulation of TEM data **with IP**

EMTDIP code
(Marchant et al., 2015)

- Maxwell's equations:

Frequency domain

$$\vec{\nabla} \times \vec{E} = -\imath\omega\vec{B}$$
$$\vec{\nabla} \times \mu^{-1}\vec{B} - \vec{J} = \vec{J}_s$$

Time domain

$$\vec{\nabla} \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$$
$$\vec{\nabla} \times \mu^{-1}\vec{b} - \vec{j} = \vec{j}_s$$

Ohm's law in **frequency** domain

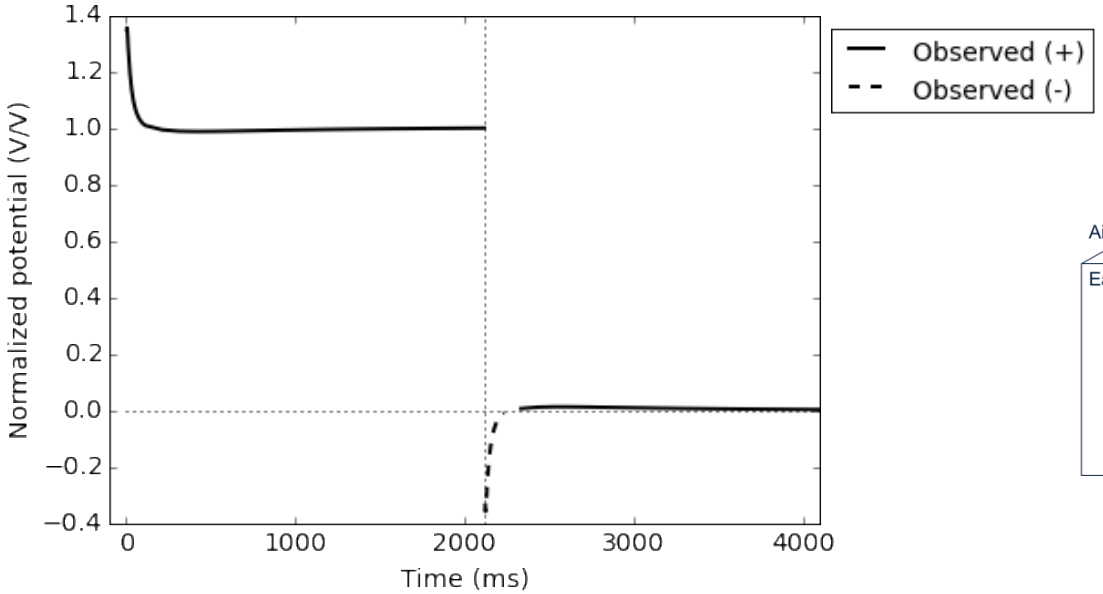
$$\vec{J} = \sigma(\omega)\vec{E}$$

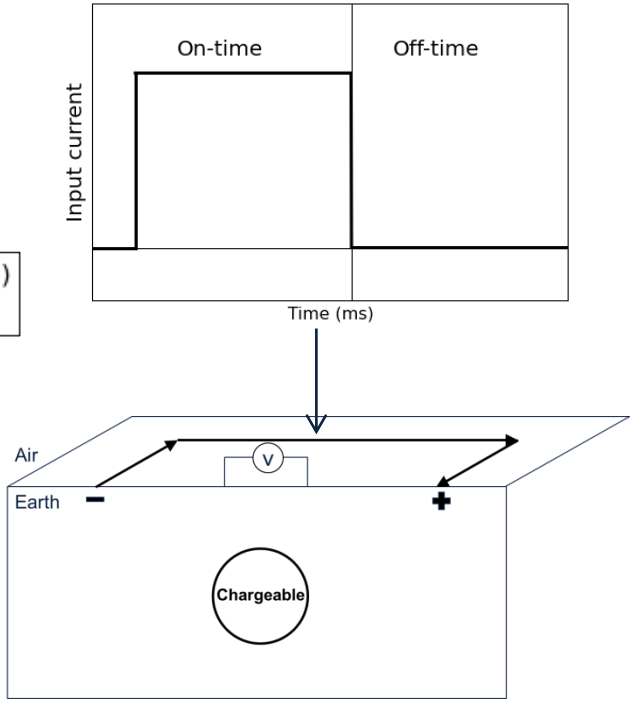
Ohm's law in **time** domain

$$\vec{j} = \sigma(t) \otimes \vec{e}(t)$$
$$\otimes : convolution$$

Observed response?

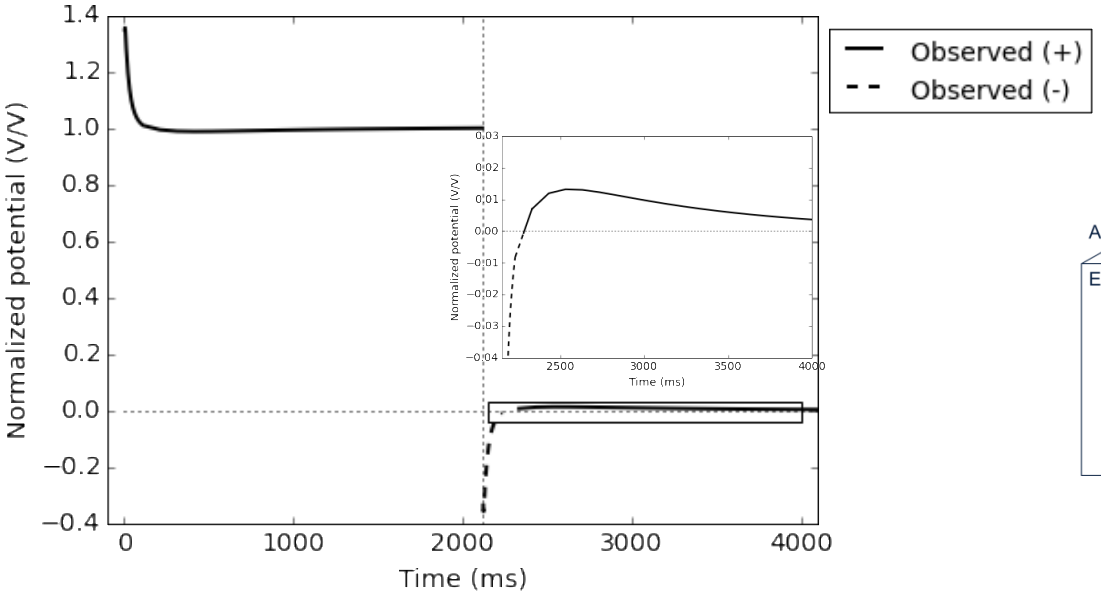
- Voltage

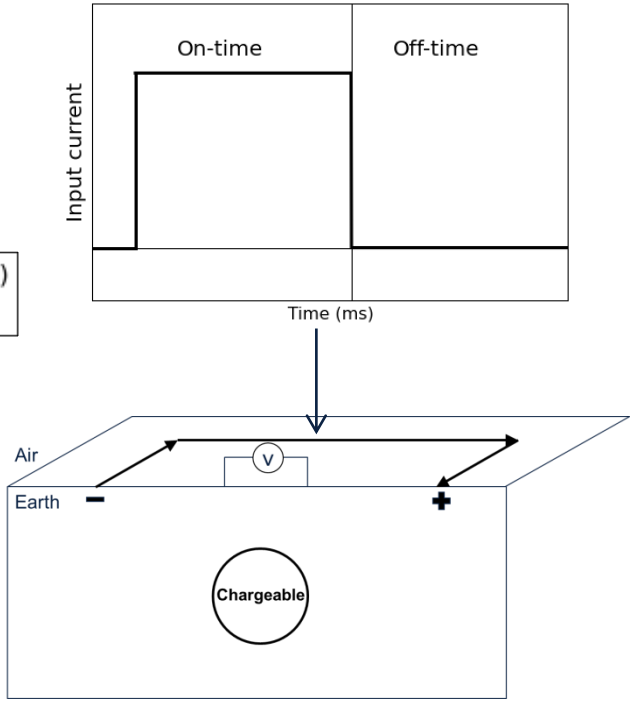




Observed response?

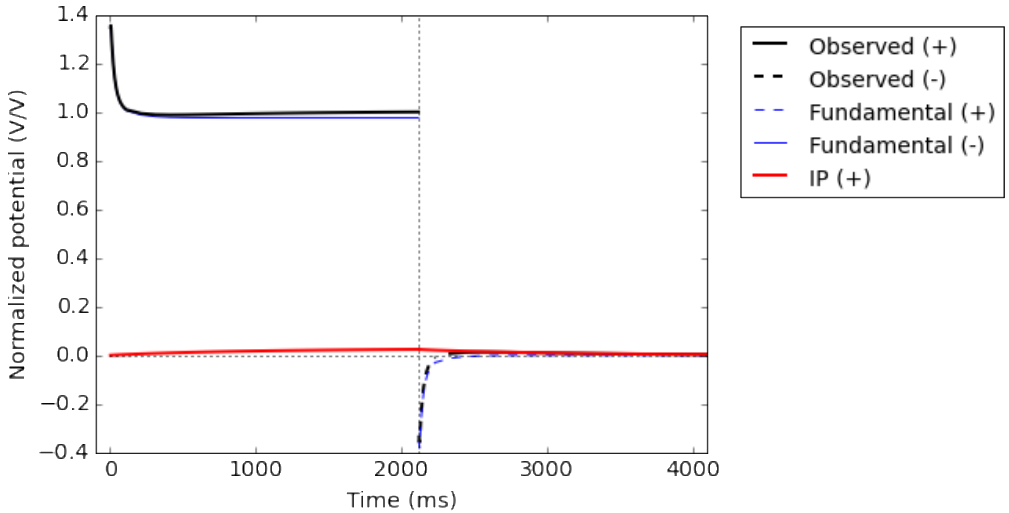
- Voltage

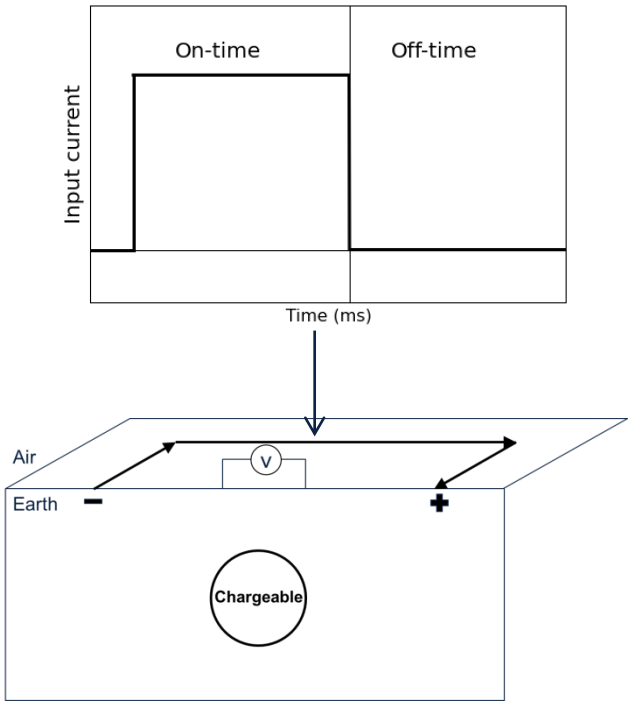




Define IP datum

- Voltage





- IP datum:

IP = Observation - Fundamental

$$d^{IP}(t) = F[\sigma(t)] - F[\sigma_\infty]$$

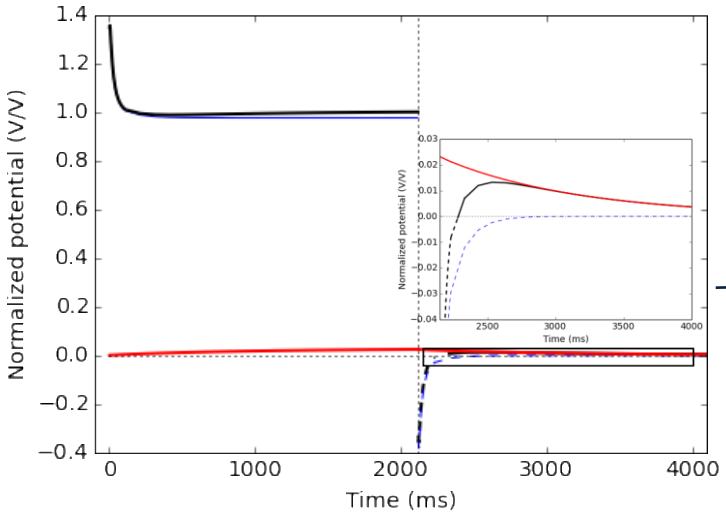
$F[\sigma(t)]$ Observation

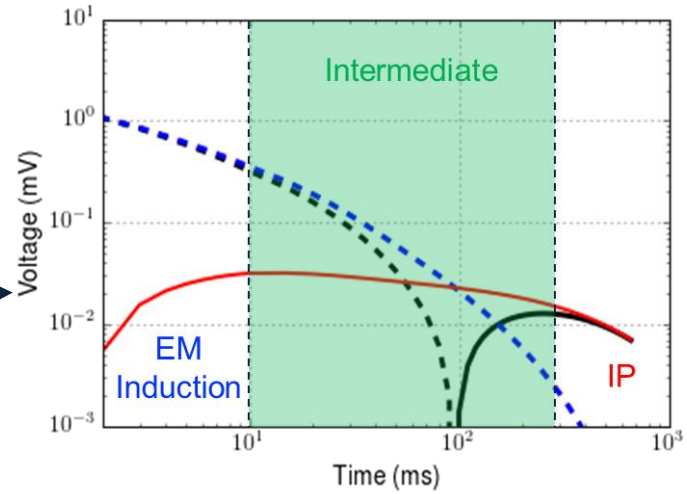
$F[\sigma_\infty]$ Fundamental

$F[\cdot]$: Maxwell's operator

Define IP datum

- Voltage





- IP datum:

IP = Observation - Fundamental

$$d^{IP}(t) = F[\sigma(t)] - F[\sigma_\infty]$$

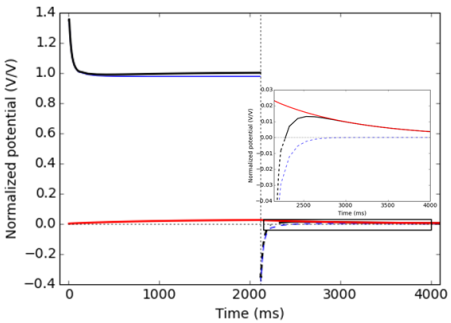
$F[\sigma(t)]$ Observation

$F[\sigma_\infty]$ Fundamental

$F[\cdot]$: Maxwell's operator

Two stage IP inversion workflow

- Oldenburg and Li (1994)



Invert DC data,
to recover σ_∞

Linearized equations

Invert d^{IP} data,
recover pseudo-chargeability

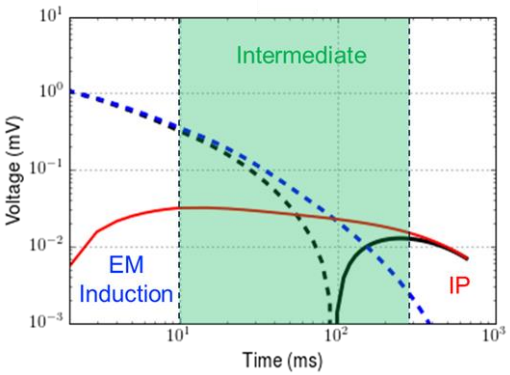
$$d^{IP}(t) = G\tilde{\eta}(t)$$

$G(\sigma_\infty)$: Sensitivity function
$\tilde{\eta}$: Pseudo-chargeability

Seigel (1959)

3D TEM-IP inversion workflow

- Kang and Oldenburg (2015)



Invert TEM data,
to recover σ_∞

Compute IP datum
Remove EM responses

Linearized equations

Invert d^{IP} data,
recover pseudo-chargeability

Estimate intrinsic
IP parameters

IP = Observation - Fundamental

$$d^{IP}(t) = F[\sigma(t)] - F[\sigma_\infty]$$

$F[\cdot]$: Maxwell's operator

$$d^{IP}(t) = G\tilde{\eta}(t)$$

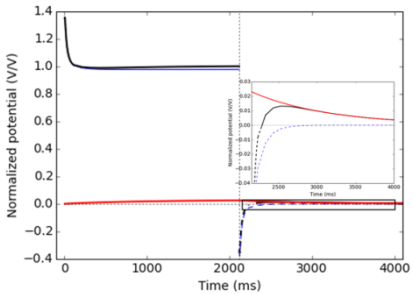
$G(\sigma_\infty)$: Sensitivity function
$\tilde{\eta}$: Pseudo-chargeability

An objective

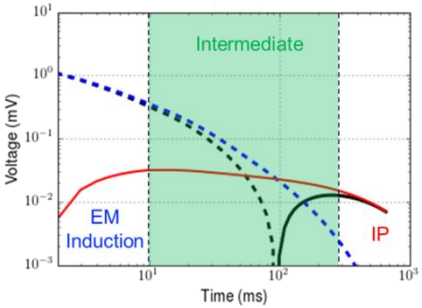
- Decouple EM induction effects in the observed data

IP = Observation - Fundamental

$$d^{IP}(t) = F[\sigma(t)] - F[\sigma_\infty]$$

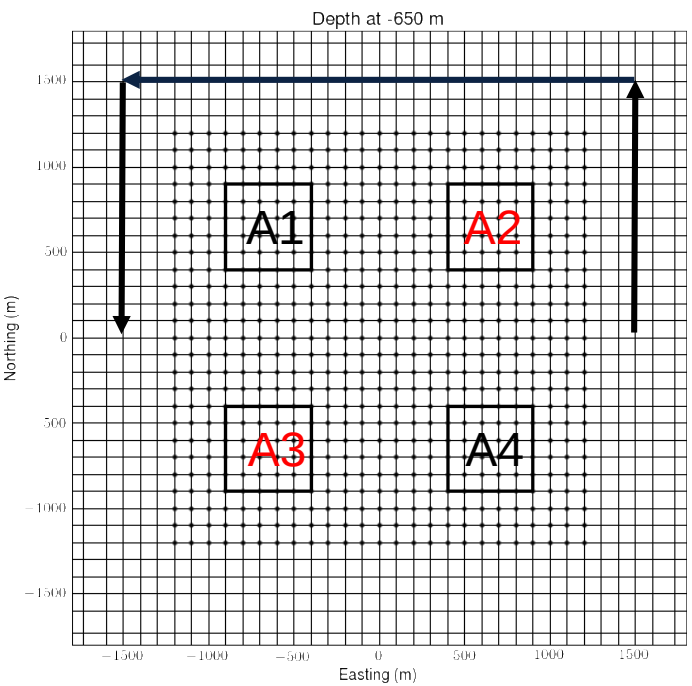


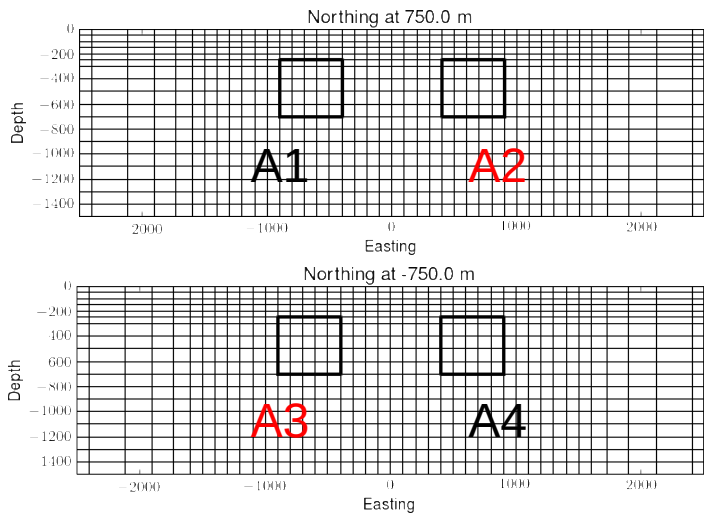
  - What conductivity?
    - Half-space σ_∞^{half}
    - Recovered σ_{est}^{DC} by inverting DC data,
    - Recovered σ_{est}^{EM} by inverting TEM data



- Invert IP datum, recover 3D chargeability

Forward modelling set up





3D mesh:

- Core cells: 100mx100mx50m 100mx100mx100m
- # of cells: 52x52x55

Measure potential difference (Easting direction)

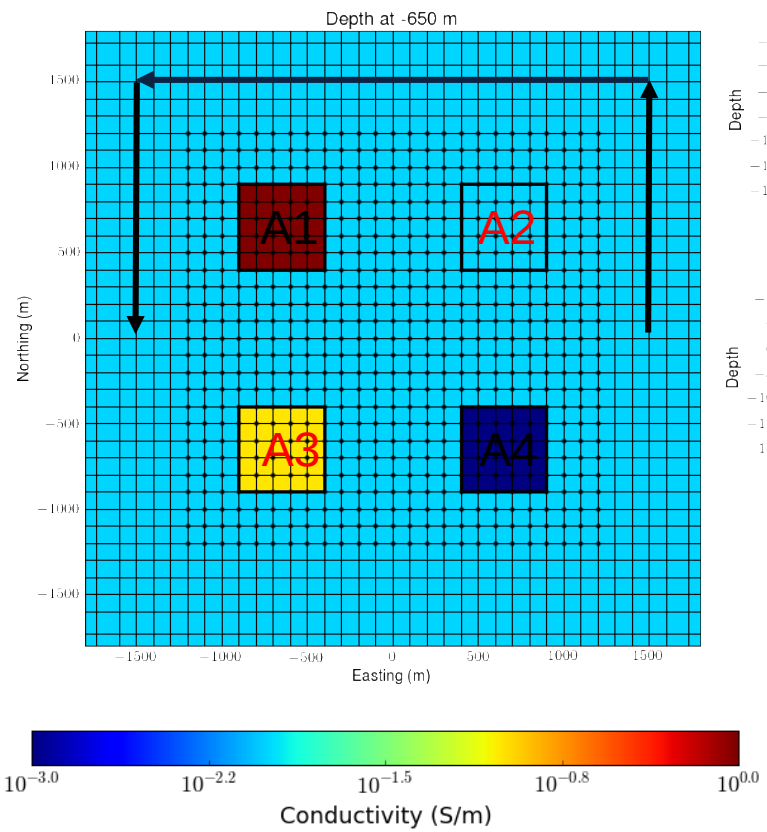
- 200 m bi-pole (625 mid points)

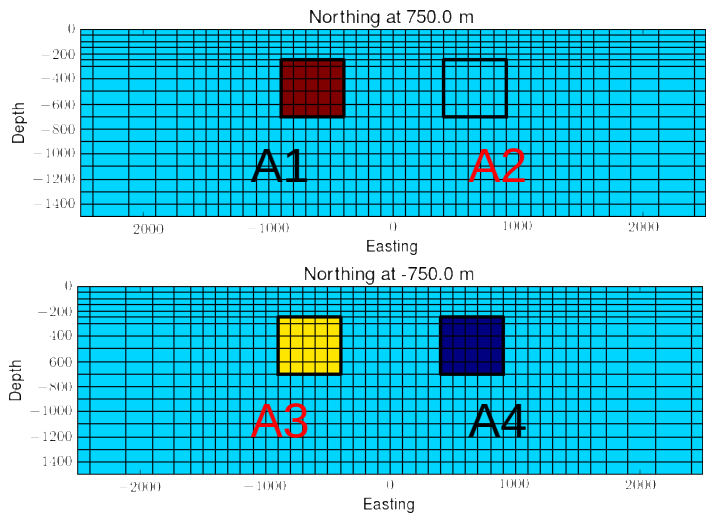
Time range: 1 – 600 ms

Chargeable objects: A2 and A3

- Cole-Cole parameters: $\eta = 0.1$
  $\tau = 0.5$ s
  $c = 1$

Forward modelling set up





Conductivity at Infinite frequency:

$$\sigma_\infty^{half} = 0.01 \text{ S/m}$$
$$\sigma_\infty^{A1} = 1 \text{ S/m}$$
$$\sigma_\infty^{A2} = 0.01 \text{ S/m}$$
$$\sigma_\infty^{A3} = 0.1 \text{ S/m}$$
$$\sigma_\infty^{A4} = 0.001 \text{ S/m}$$

Measure potential difference (Easting direction)

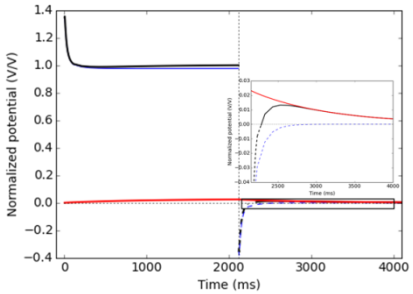
- 200 m bi-pole (625 mid points)

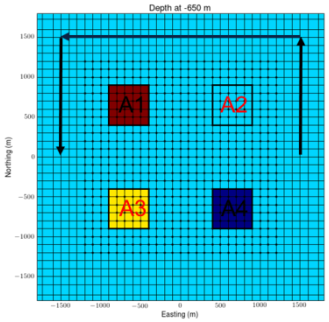
Time range: 1 – 600 ms

Chargeable objects: A2 and A3

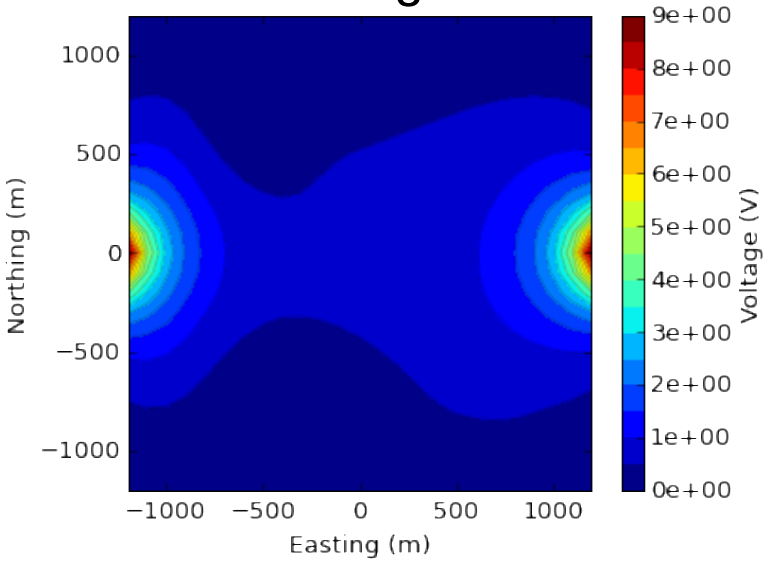
- Cole-Cole parameters: $\eta = 0.1$
  $\tau = 0.5$ s
  $c = 1$

Observed DC data

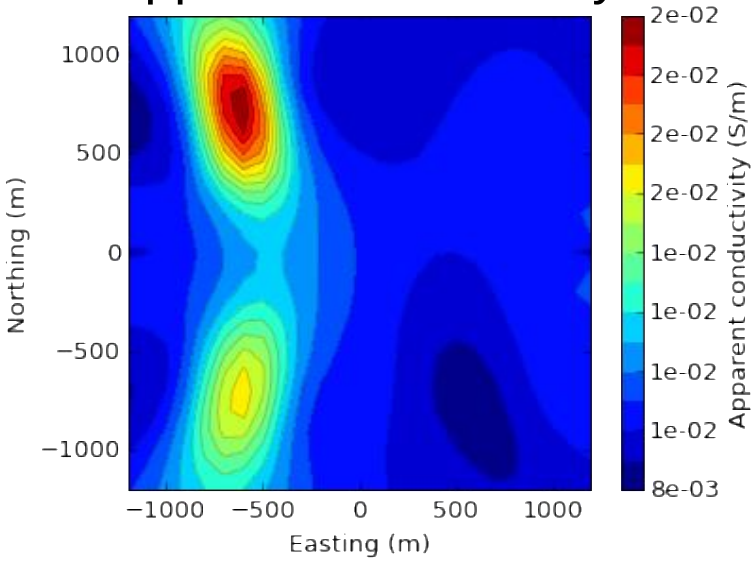




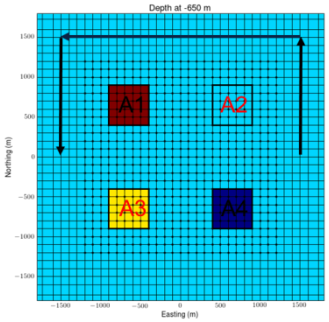
Voltage



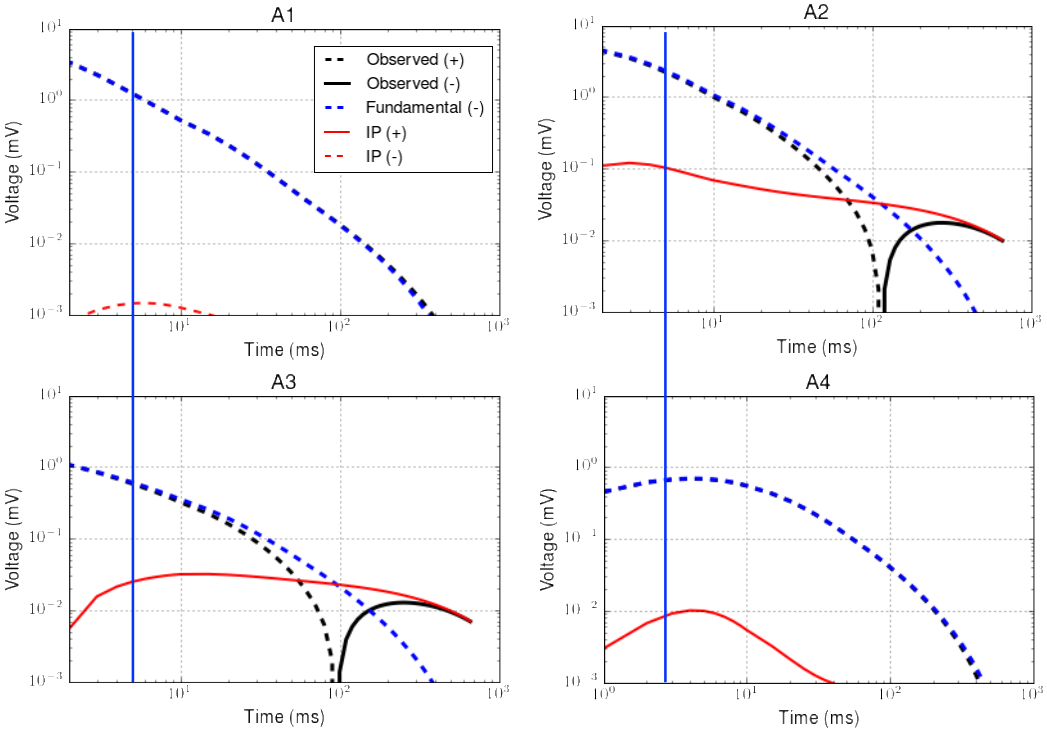
Apparent conductivity



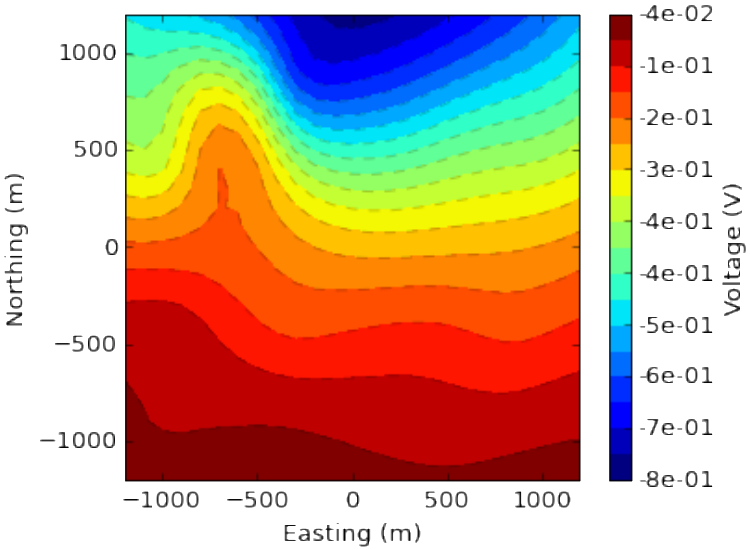
Observed data (off-time)



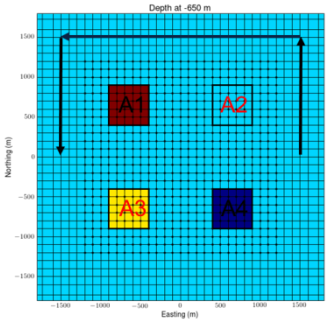
- Decaying curves at A1-A4



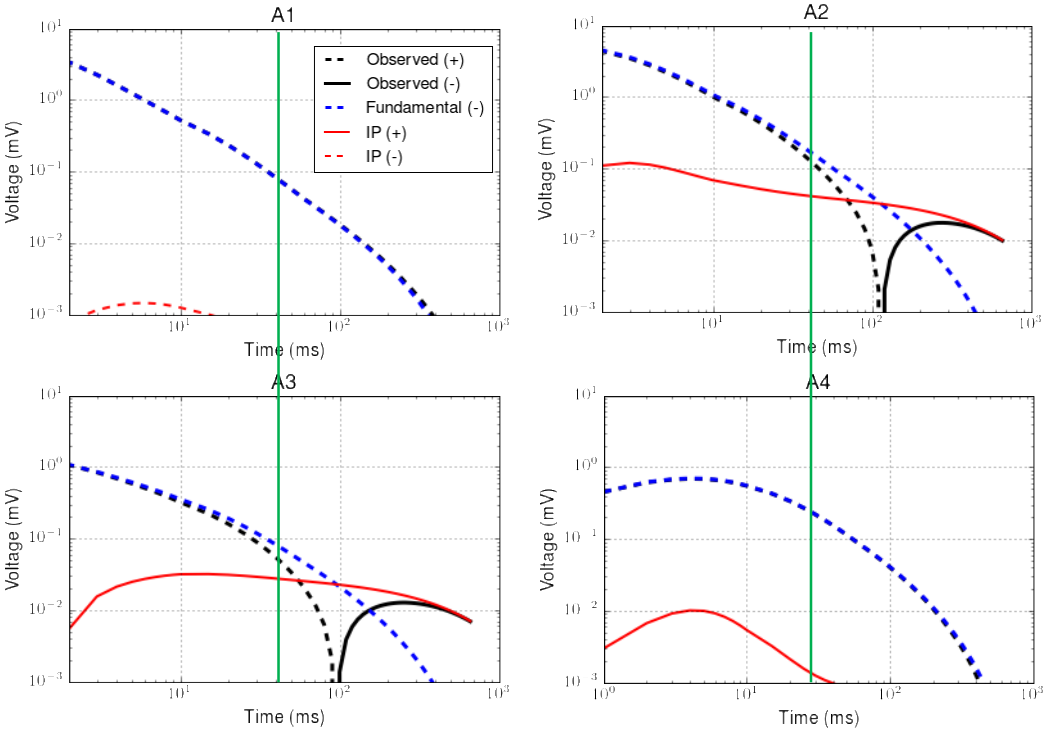
Off-time at 5 ms

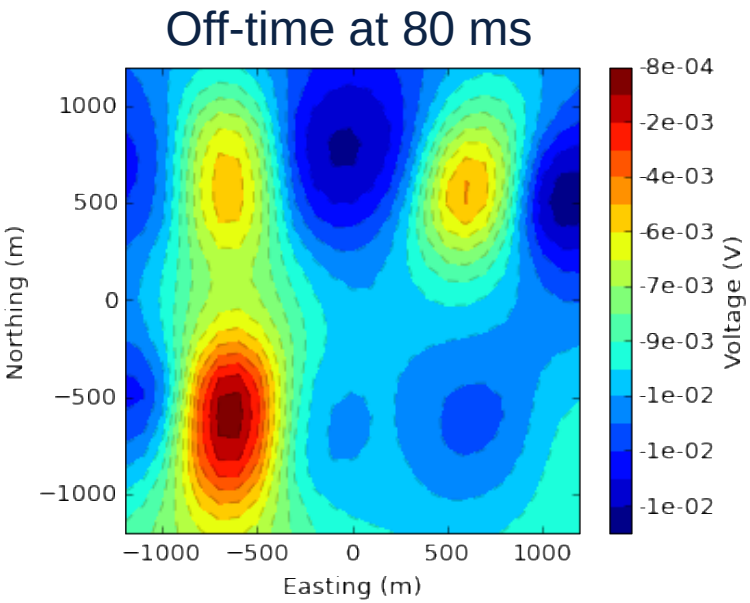


Observed data (off-time)

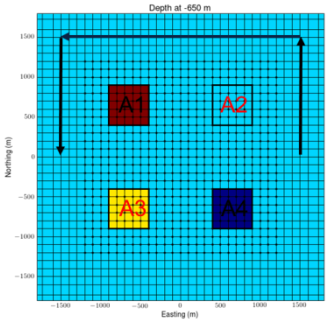


- Decaying curves at A1-A4

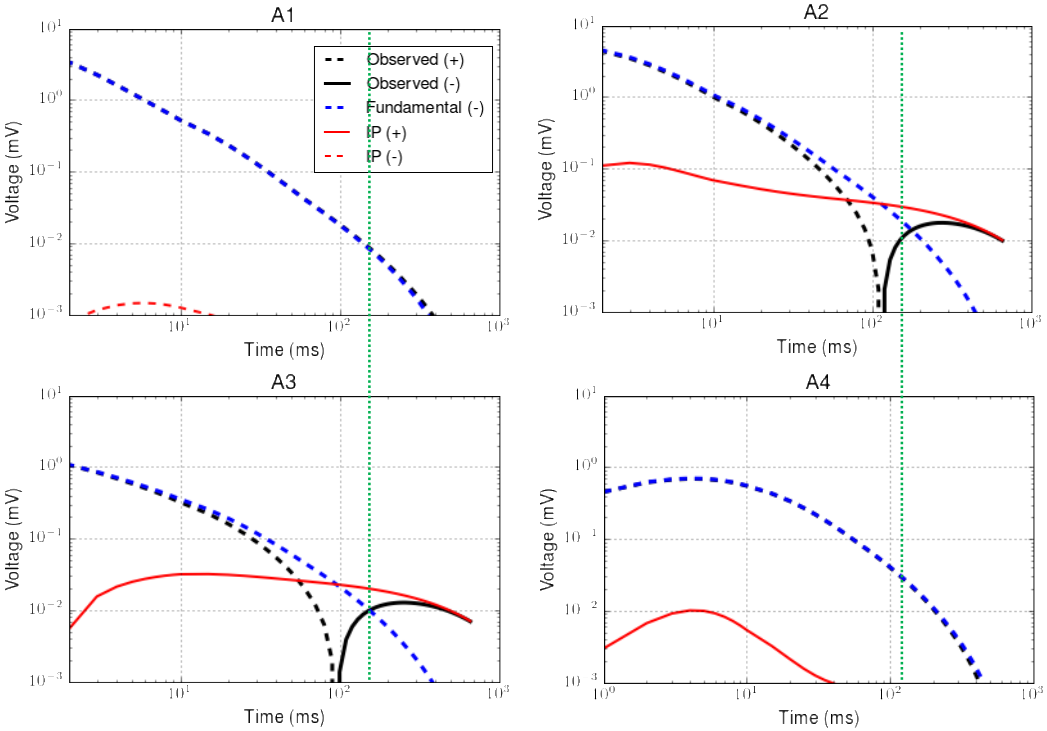




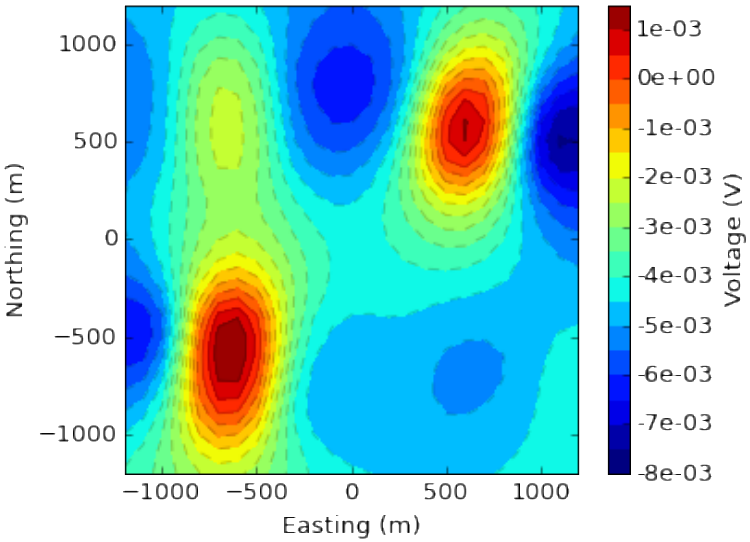
Observed data (off-time)



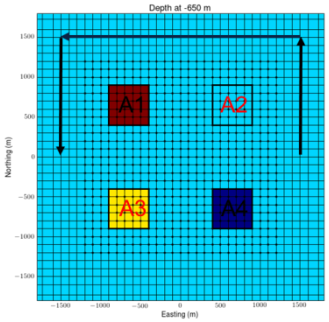
- Decaying curves at A1-A4



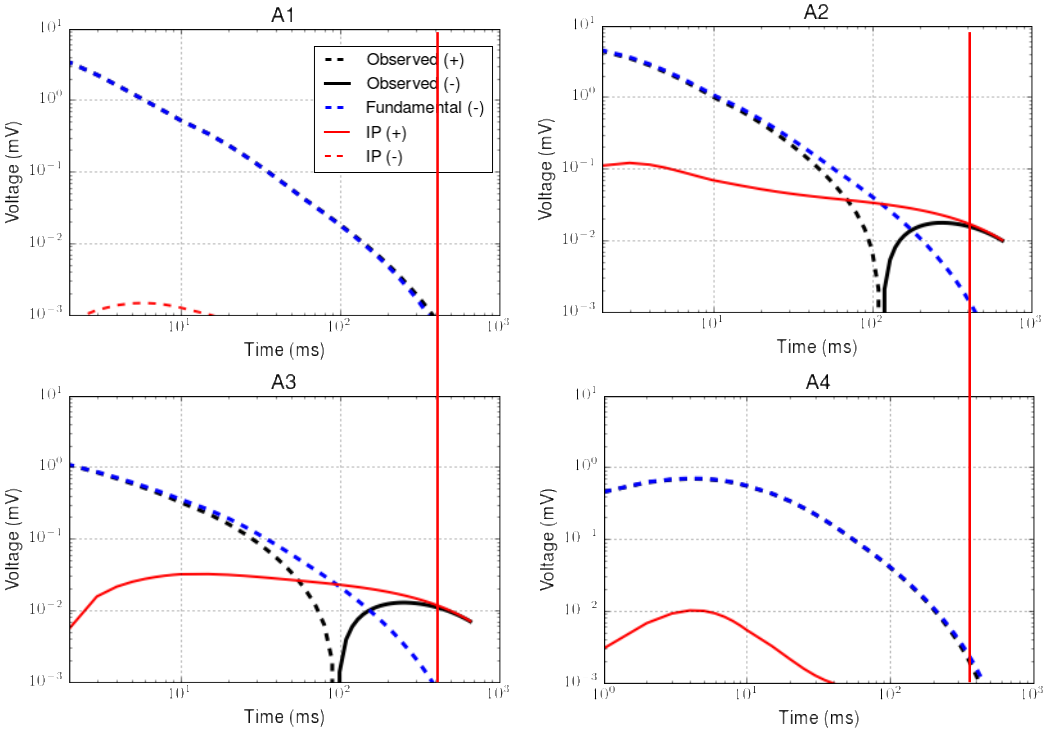
Off-time at 130 ms

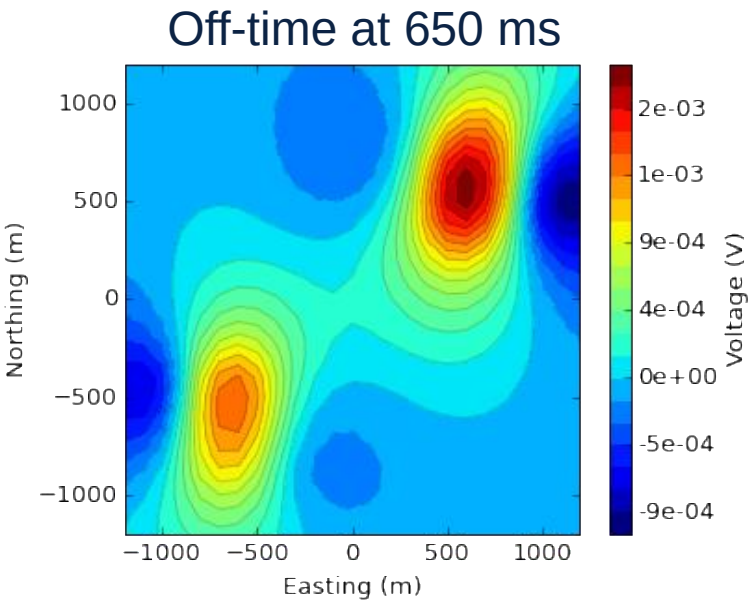


Observed data (off-time)

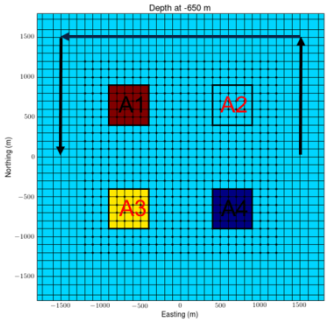


- Decaying curves at A1-A4

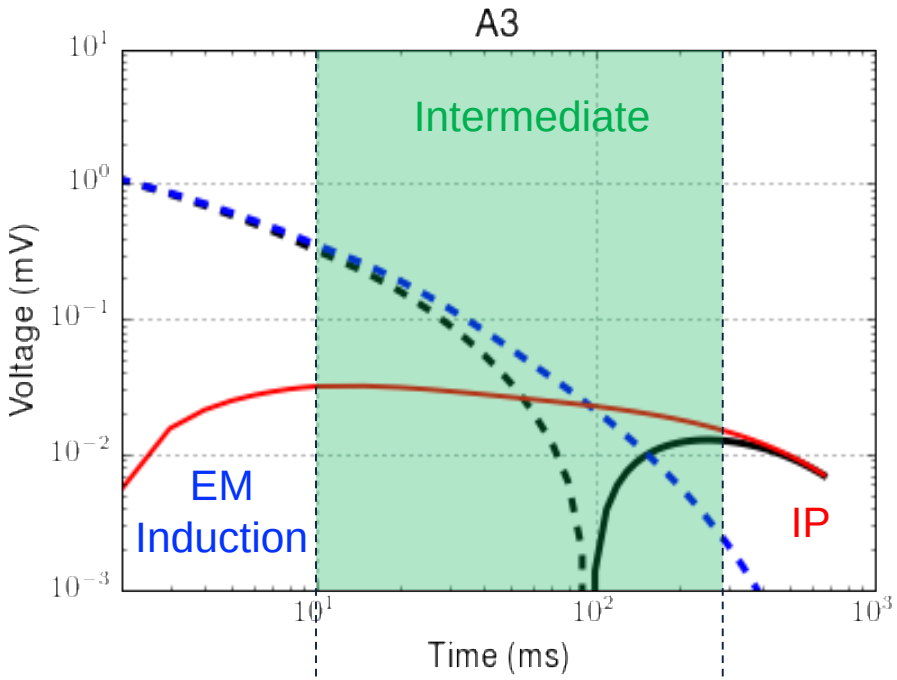


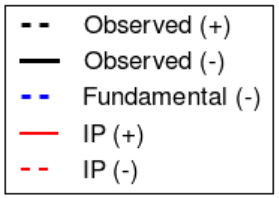


EM decoupling



- Time decaying curves (off-time)



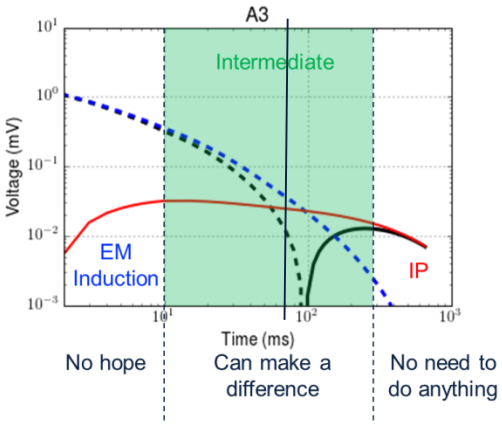


IP = Observation - Fundamental

$$d^{IP}(t) = F[\sigma(t)] - F[\sigma_\infty]$$

No hope | Can make a difference | No need to do anything

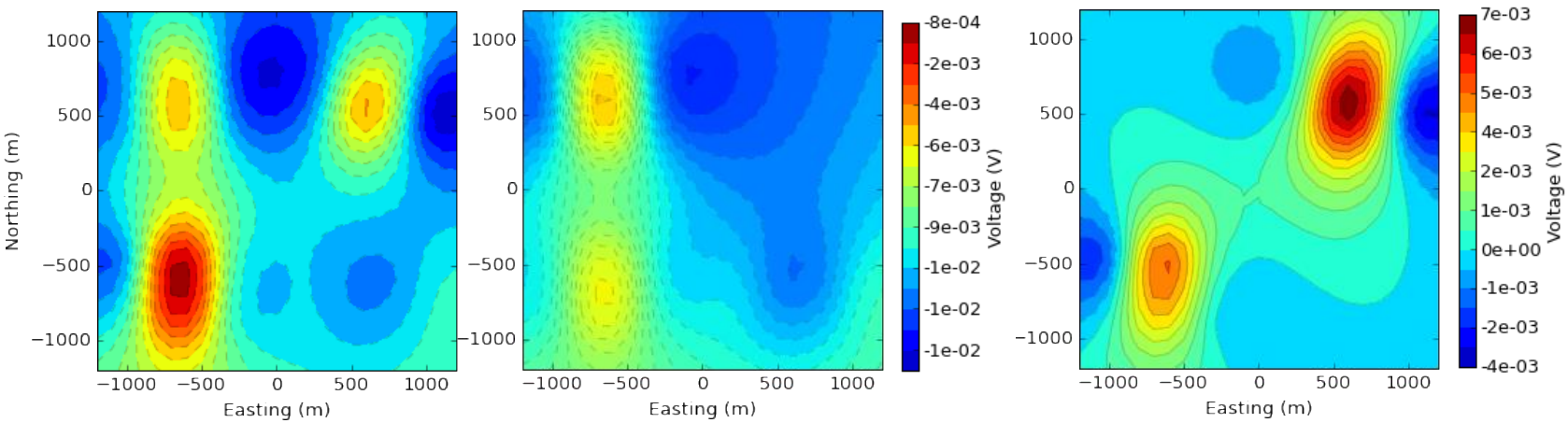
EM decoupling: true σ_∞



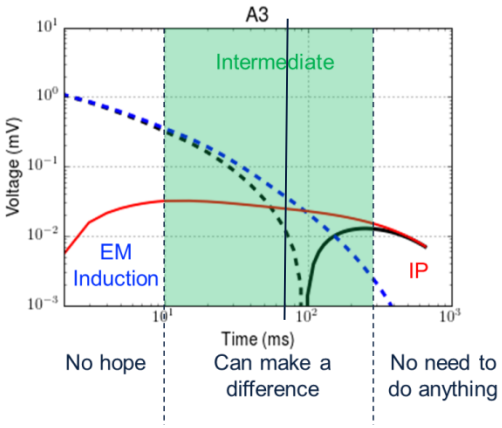
- Off-time at 80 ms

$$d^{IP}(t) = F[\sigma(t)] - F[\sigma_\infty]$$

Observation Fundamental IP



EM decoupling: true σ_{∞}^{half}



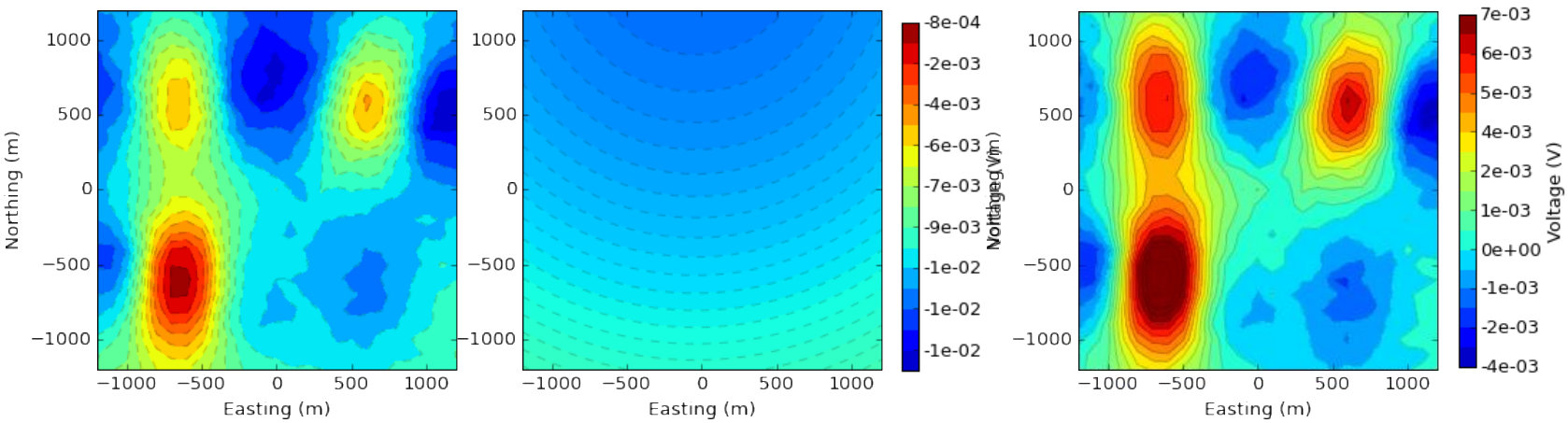
- Off-time at 80 ms

$$d_{raw}^{IP}(t) = F[\sigma(t)] - F[\sigma_{\infty}^{half}] + noise(t)$$

Observation

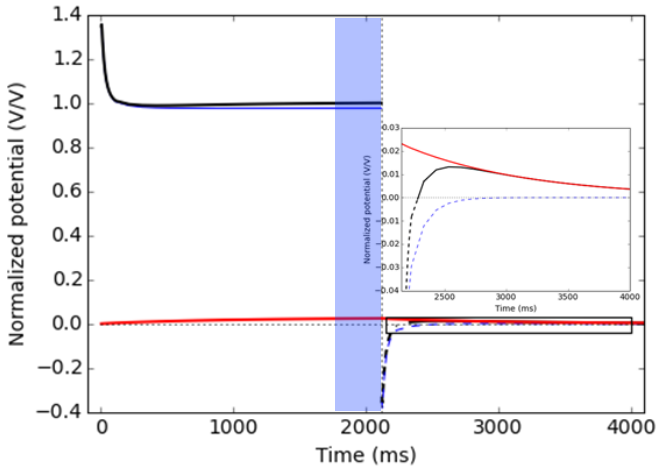
Fundamental

IP

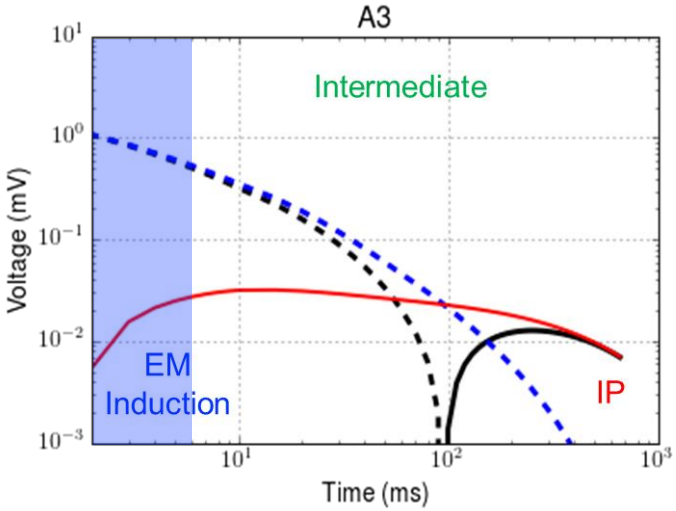


What data we have?

Late on-time data (DC)



Early off-time data (TEM)



3D inversion methodology

- Data misfit:

$$\phi_d = \|\mathbf{W_d}(\mathbf{Am} - d^{obs}|)\|_2^2 = \sum_{j=1}^{N}(\frac{\mathbf{d}_j^{pred} - \mathbf{d}_j^{obs}}{\epsilon_j}),$$

- Tikhanov style regularization:

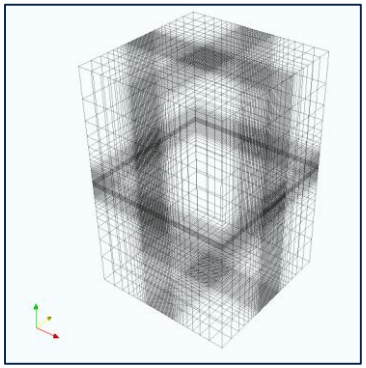
$$\phi_m = \alpha_s\|\mathbf{W}_s\mathbf{W}(\mathbf{m} - \mathbf{m}_{ref})\|_2^2 + \alpha_x\|\mathbf{W}_x\mathbf{W}(\mathbf{m} - \mathbf{m}_{ref})\|_2^2 + \alpha_y\|\mathbf{W}_y\mathbf{W}(\mathbf{m} - \mathbf{m}_{ref})\|_2^2 + \alpha_z\|\mathbf{W}_z\mathbf{W}(\mathbf{m} - \mathbf{m}_{ref})\|_2^2,$$

- Statement of the inversion:

$$minimize\ \phi = \phi_d(\mathbf{m}) + \phi_m(\mathbf{m})$$

- Depth weight:

$$\mathbf{W} = \mathbf{diag}\Big((z - z_0)^{\alpha}\Big)$$



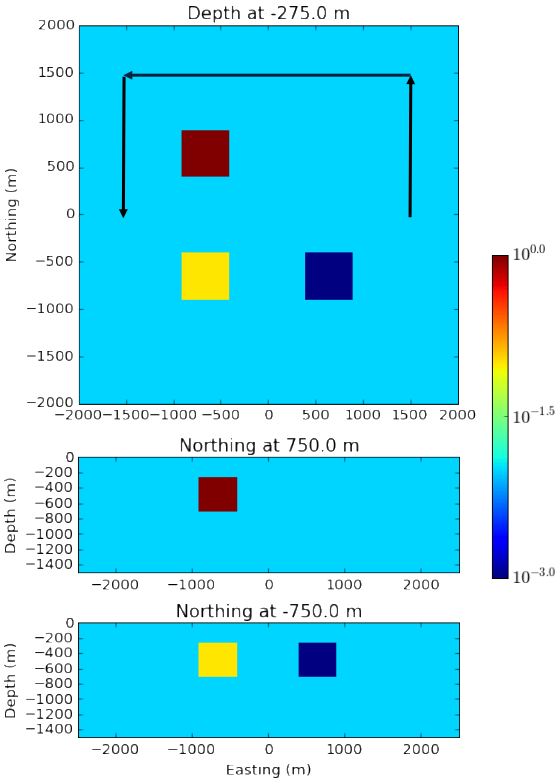
Simulation and Parameter Estimation in Geophysics

DC-IP inversion: SimPEG-DCIP
TEM inversion: UBC-H3DTD code

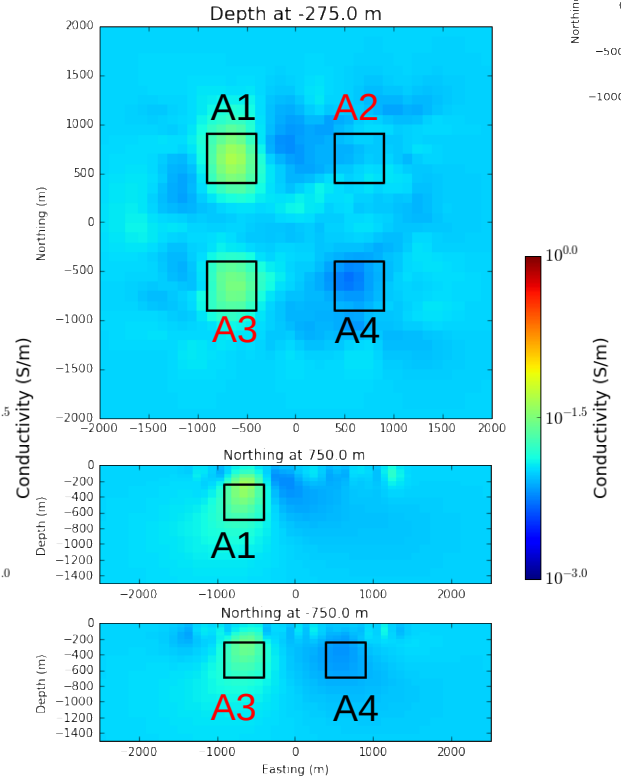
3D DC inversion

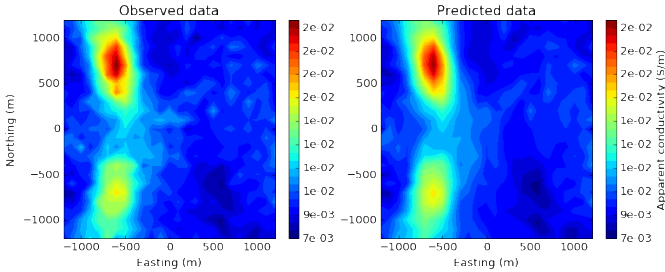
- Recover 3D conductivity

True σ_∞



Estimated $\sigma_\infty (= \sigma_{est}^{DC})$

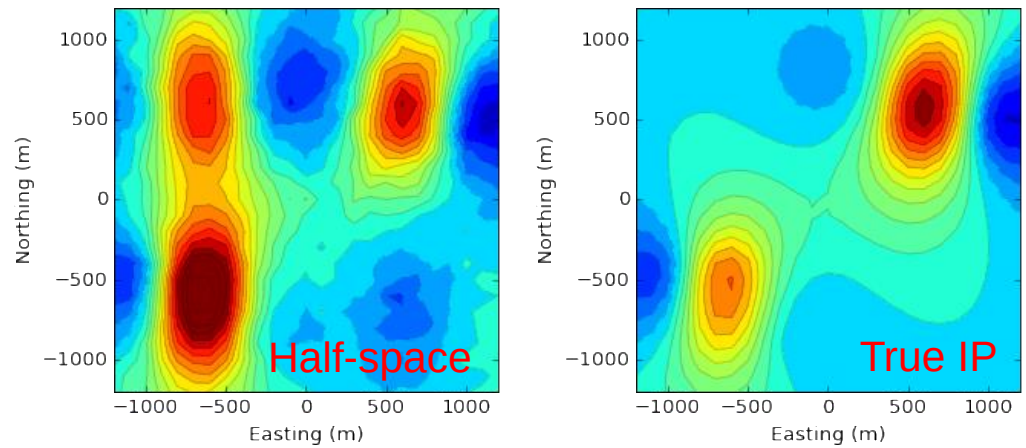




Depth weight:

$$\mathbf{W} = \mathbf{diag}\Big((z - z_0)^\alpha\Big)$$

EM decoupling: σ_{est}^{DC}



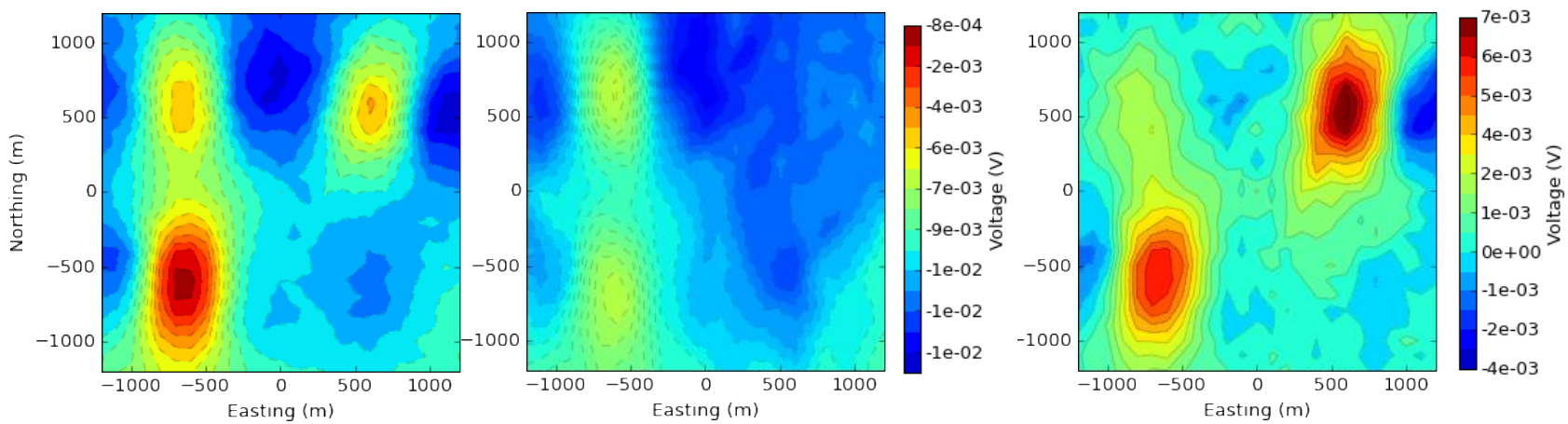
- Off-time at 130 ms

$$d_{raw}^{IP}(t) = F[\sigma(t)] - F[\sigma_{est}^{DC}] + noise(t)$$

Observation

Predicted

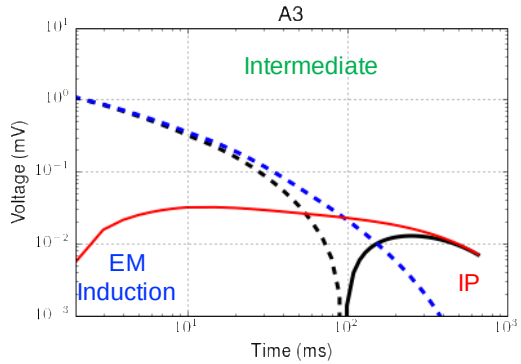
Raw IP



3D TEM inversion

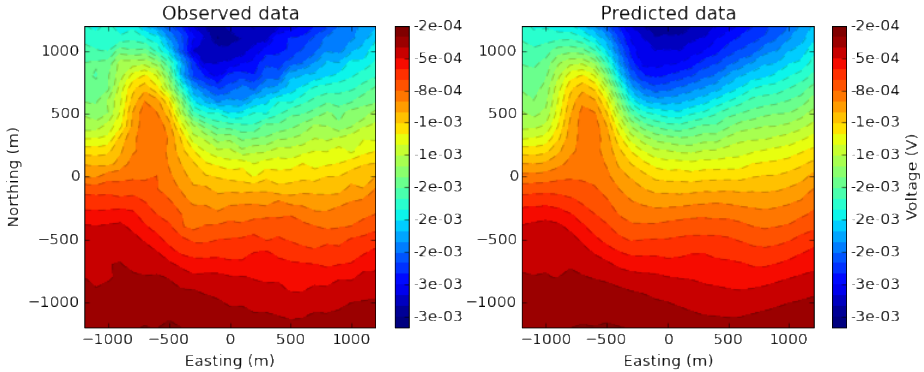
- Recover 3D conductivity

Use uncontaminated EM data

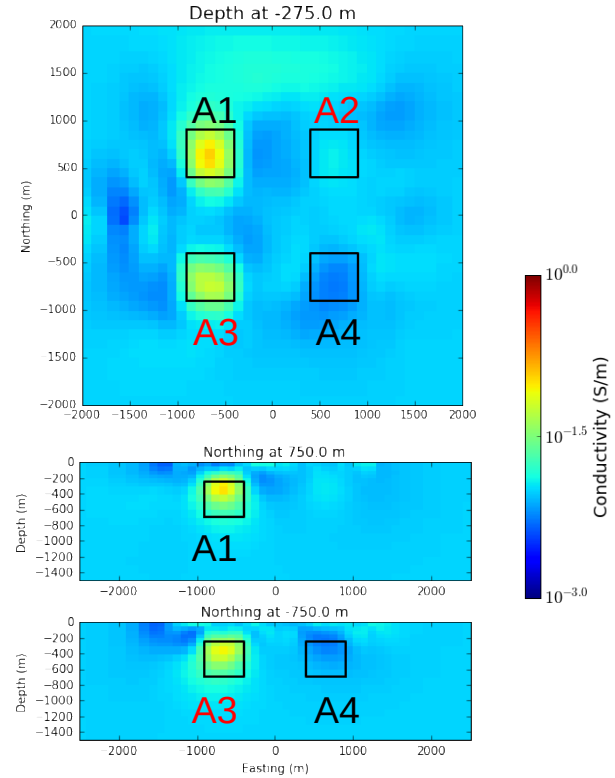


Time range: 1-6 ms
(6 channels)

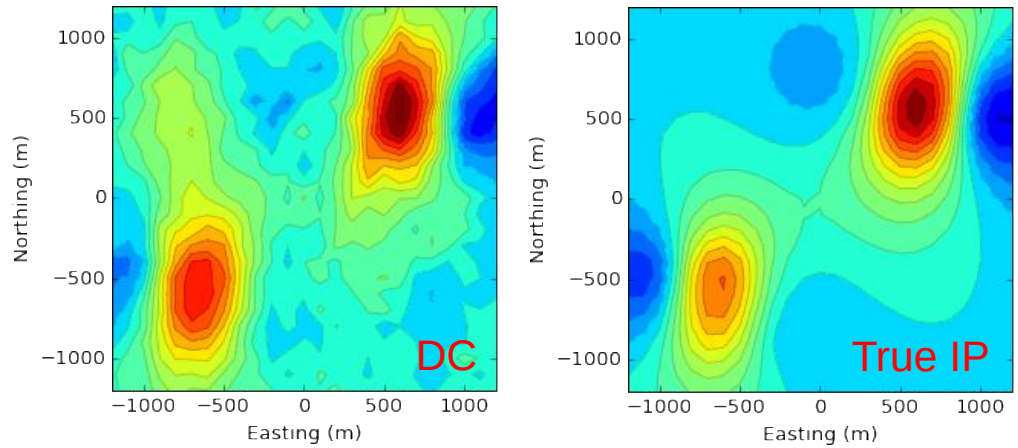
Observed vs. Predicted



Estimated $\sigma_\infty (= \sigma_{est}^{EM})$



EM decoupling: σ_{est}^{EM}



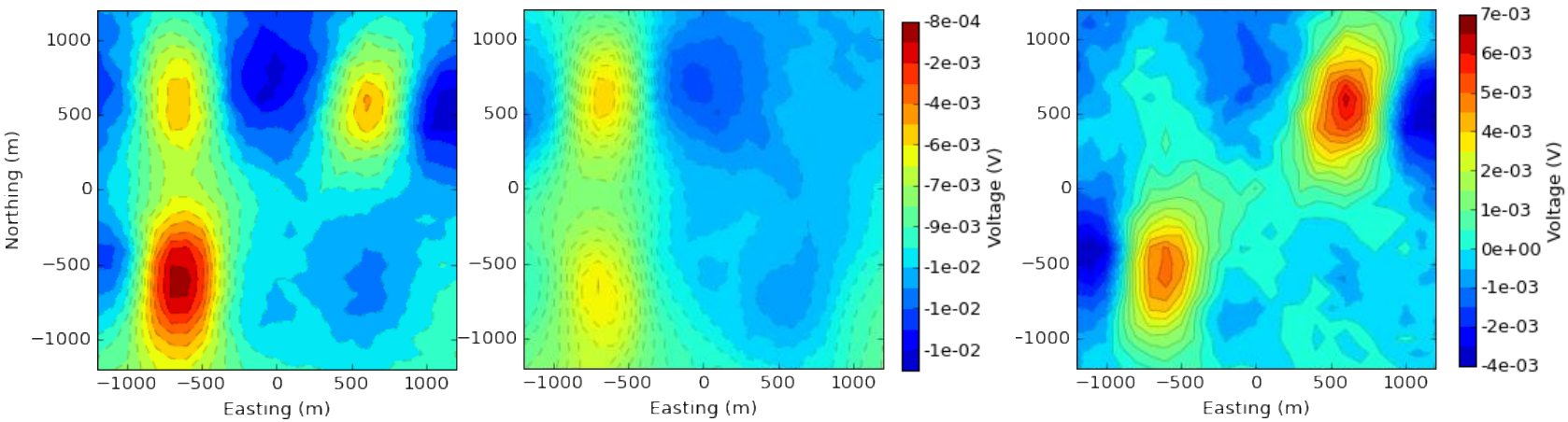
- Off-time at 80 ms

$$d_{raw}^{IP}(t) = F[\sigma(t)] - F[\sigma_{est}^{EM}] + noise(t)$$

Observation

Predicted

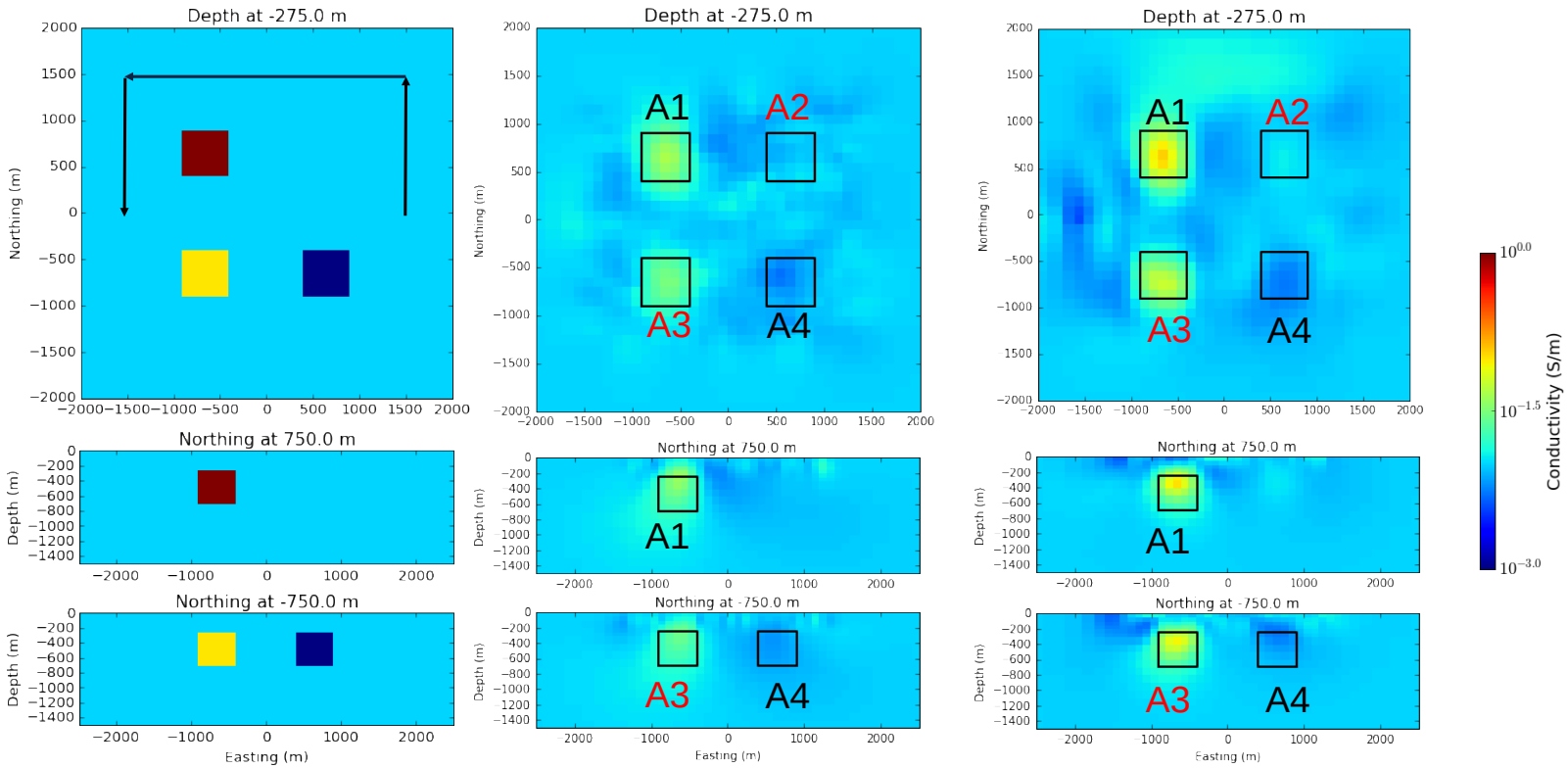
Raw IP



Comparison of 3D conductivities

- Recovered 3D conductivity

True σ_∞ Estimated $\sigma_\infty (= \sigma_{est}^{DC})$ Estimated $\sigma_\infty (= \sigma_{est}^{EM})$

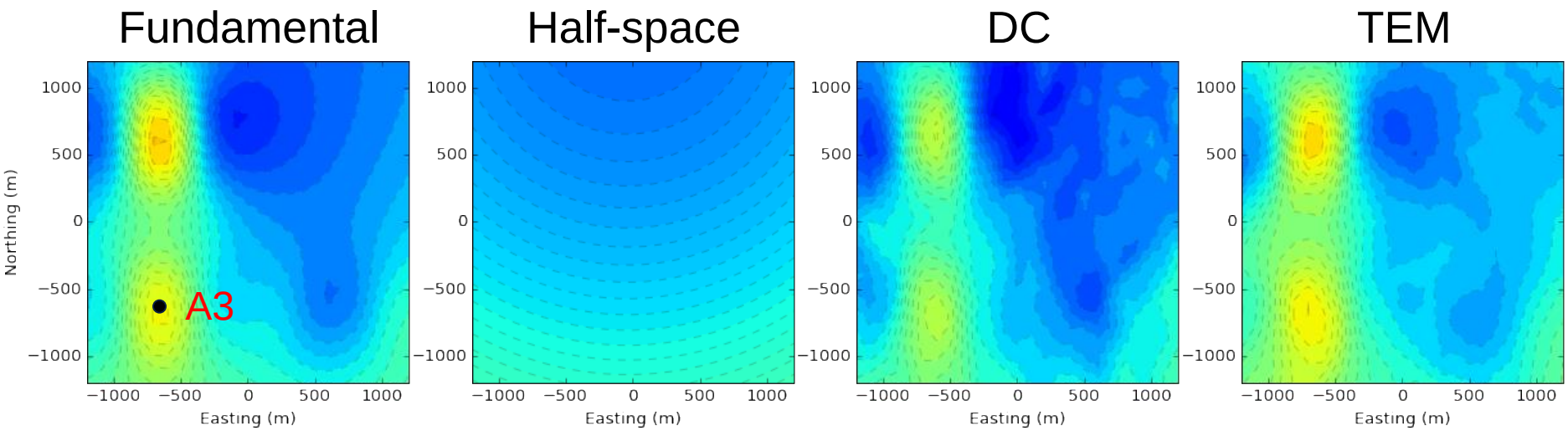


Comparisons of Fundamental

- Fundamental data at 80 ms

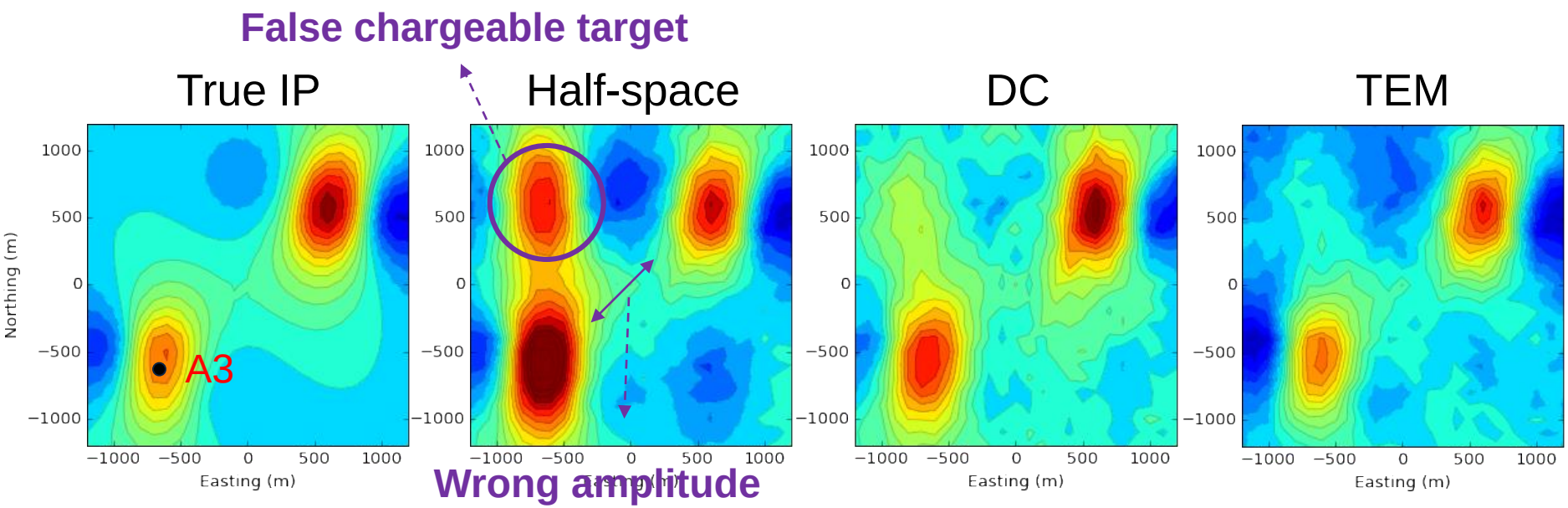
IP = Observation - Fundamental

$$d^{IP}(t) = F[\sigma(t)] - F[\sigma_\infty]$$



Comparisons of IP

- IP data at 80 ms



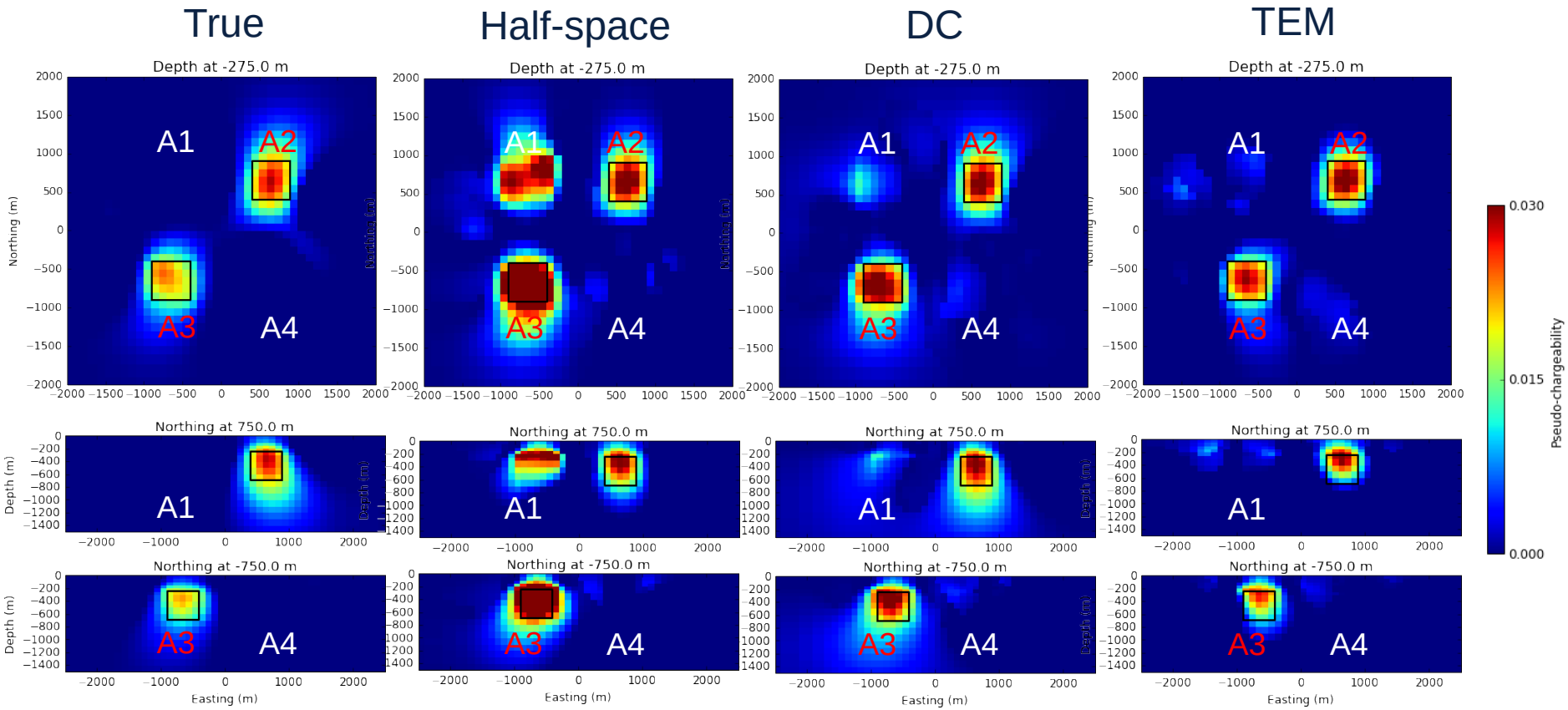
3D IP inversion

SimPEG-DCIP code

Depth weight:

$$\mathbf{W} = \mathbf{diag}\left((z - z_0)^{\alpha}\right)$$

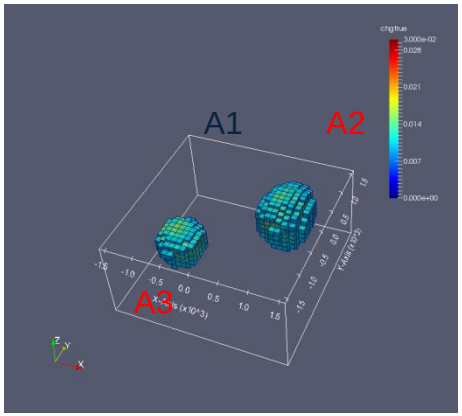
- Chargeability: recovered by inverting:



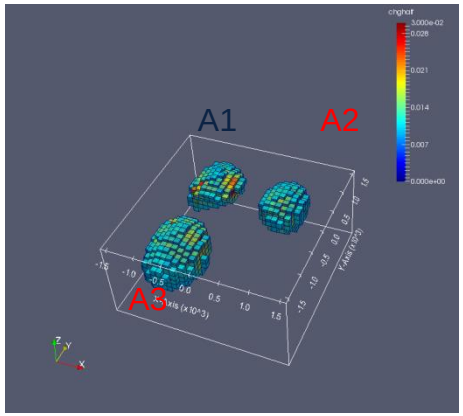
3D cut-off volume

- Pseudo-chargeability > 0.015

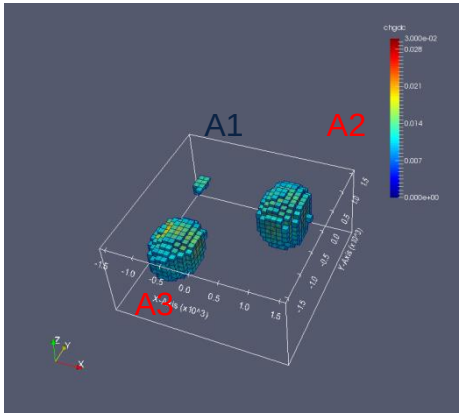
True



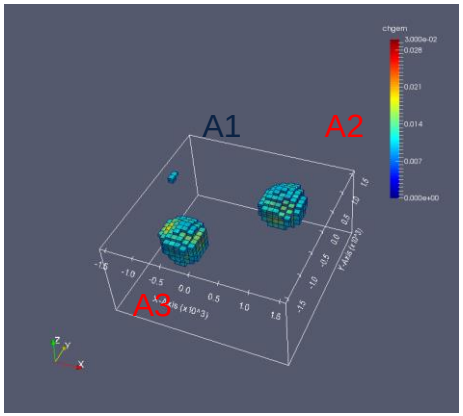
Half-space



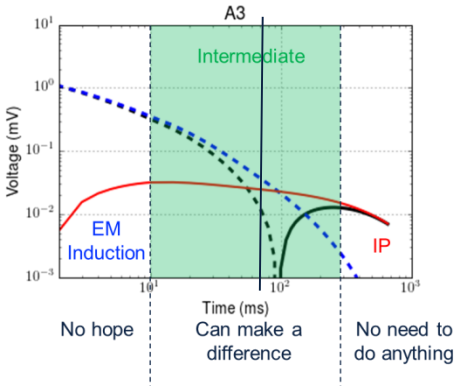
DC



TEM



Summary



3D-TEM IP inversion workflow

Invert TEM data,
to recover σ_∞

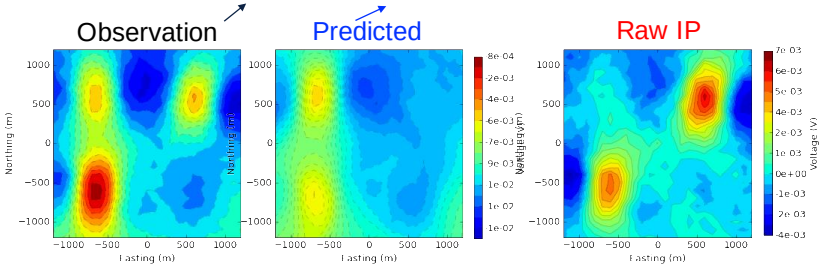
Compute IP datum
Remove EM responses

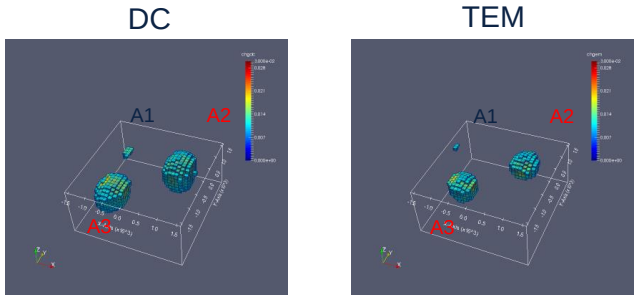
Linearized equations

Invert d^{IP} data,
recover pseudo-chargeability

Estimate intrinsic
IP parameters

$$d^{IP}_{raw}(t) = F[\sigma(t)] - F[\sigma^{EM}_{est}] + noise(t)$$

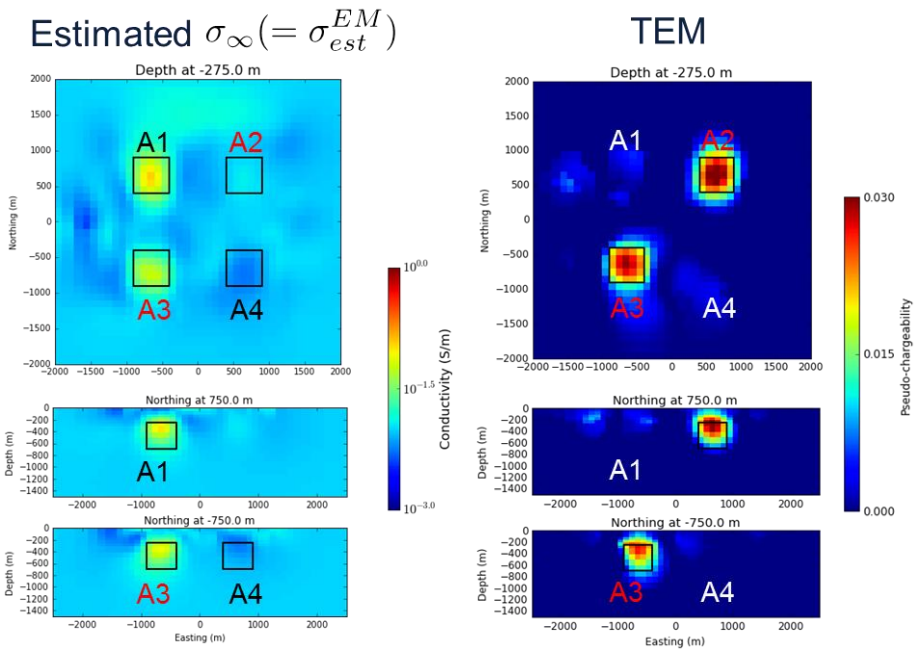




Take home

"Traditionally, **early time TEM data** has been discarded"

"By using these discarded **TEM** signals we can better estimate both **3D conductivity** and **chargeability"**



Thank you ☺

Acknowledgements

- Special thanks to
  - SimPEG developers for utility codes to make this study, and constructive discussions
  - SimPEG-DCIP package for DC-IP inversions
  - UBC-GIF for allowing us to use H3DTD codes
  - Patrick Belliveau and Eldad Haber for developing time domain IP code