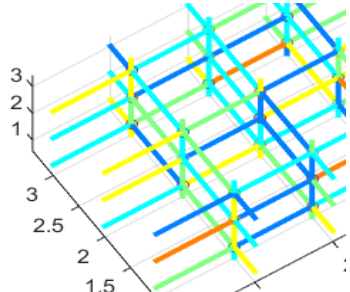
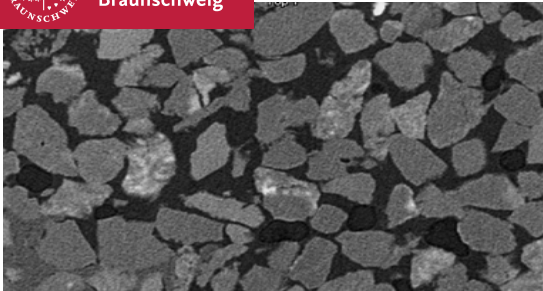




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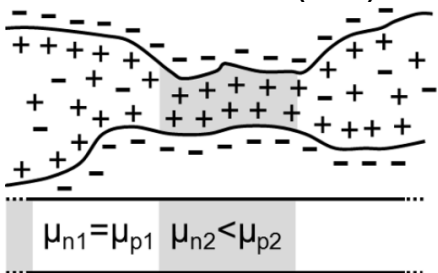
Simulation of membrane polarization for 2D and 3D pore networks

4th International Workshop on Induced Polarization

Hermann Stebner, Andreas Hördt, 07.06.2016

Membrane polarisation

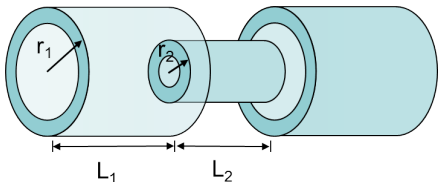
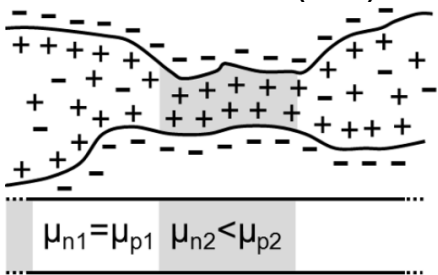
Marshall und Madden (1959)



- cations bound to mineral surface
- decreased effective anion mobility in bottlenecks

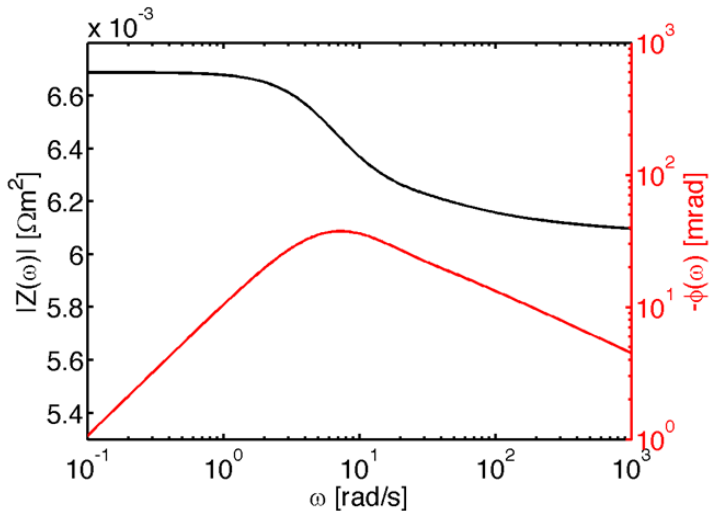
Membrane polarisation

Marshall and Madden (1959)



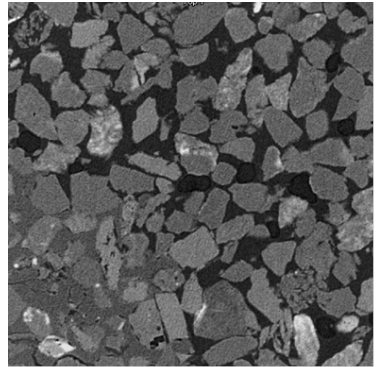
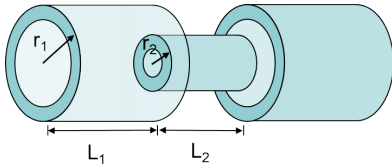
- cations bound to mineral surface
- decreased effective anion mobility in bottlenecks
- 2D-expansion (Bücker and Hördt, 2013a) with geometric parameters: r_1, r_2, L_1, L_2

Membrane polarisation



Networks

Aim

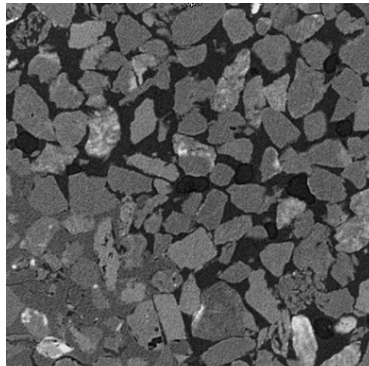
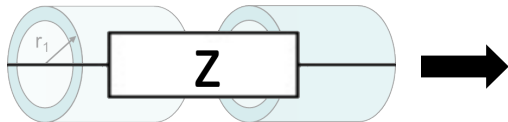


200 μm

Networks

Aim

impedance component

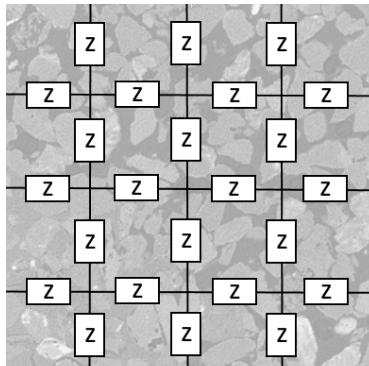
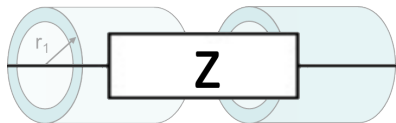


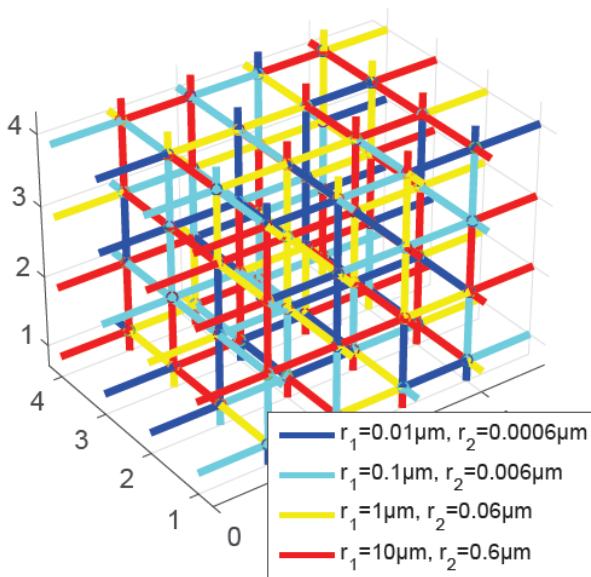
200 μm

Networks

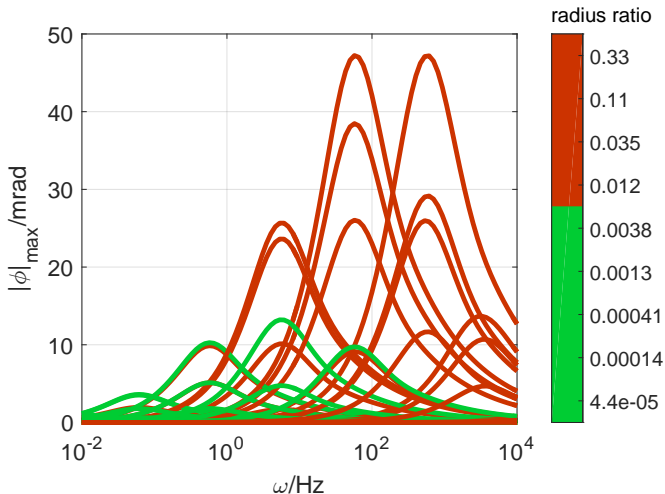
Aim

impedance component

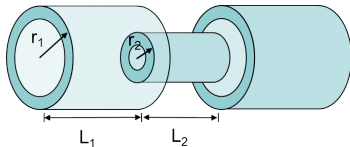




Pore combinations



Choosing geometric parameters



Pore lengths

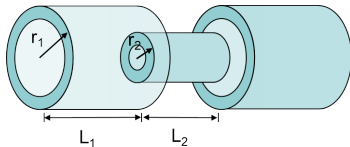
$$L_i = (c \cdot r_i)^d$$

r_i , pore radii

c , **L-r** ratio parameter

d , exponent parameter

Choosing geometric parameters



Pore lengths

$$L_i = (c \cdot r_i)^d$$

r_i , pore radii

c , **L-r** ratio parameter

d , exponent parameter

$d = 2 \rightarrow$ maximum phase shift (Hördt et. al, B11, late afternoon)

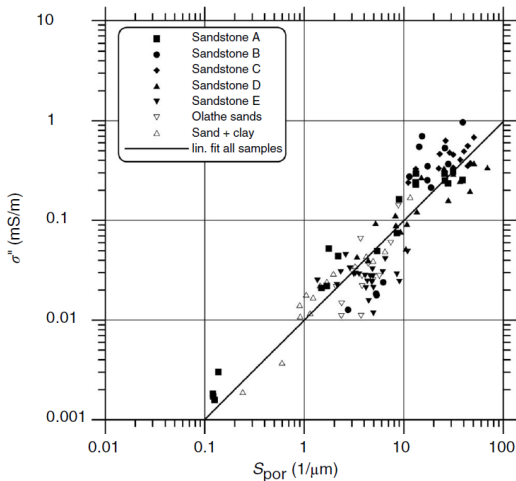
$c = ?$

σ'' vs S_{por}

Imaginary conductivity σ''
at 1Hz vs specific inner
surface S_{por} .

with $\sigma'' = a \cdot S_{por}^b$
and $b \approx 1$

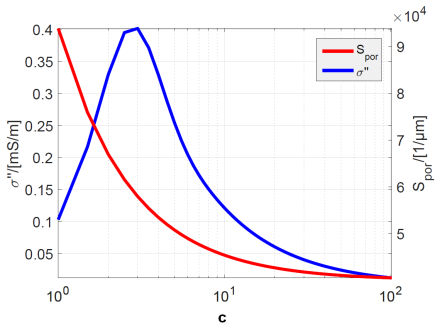
(from Weller et. al., 2010)



σ'' vs S_{por}

$$L_i = (c \cdot r_i)^d, \quad r_1 = 0.5\mu\text{m}, r_2 = 50\mu\text{m}$$

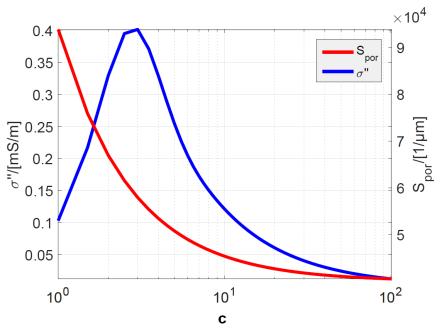
with $d=1.5$



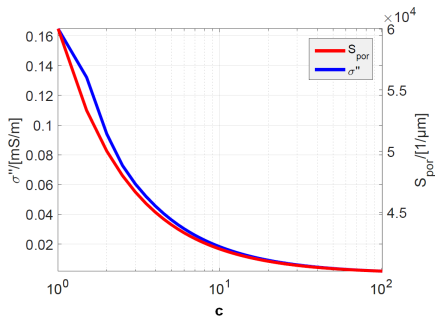
σ'' vs S_{por}

$$L_i = (c \cdot r_i)^d, \quad r_1 = 0.5\mu\text{m}, r_2 = 50\mu\text{m}$$

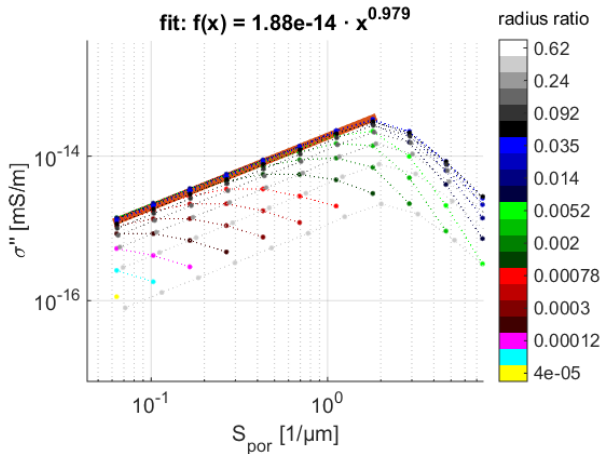
with $d=1.5$



with $d=1$

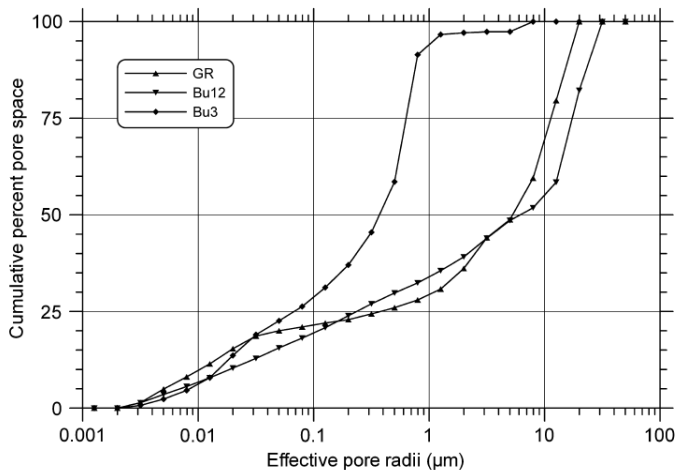


Lengths-radii relation



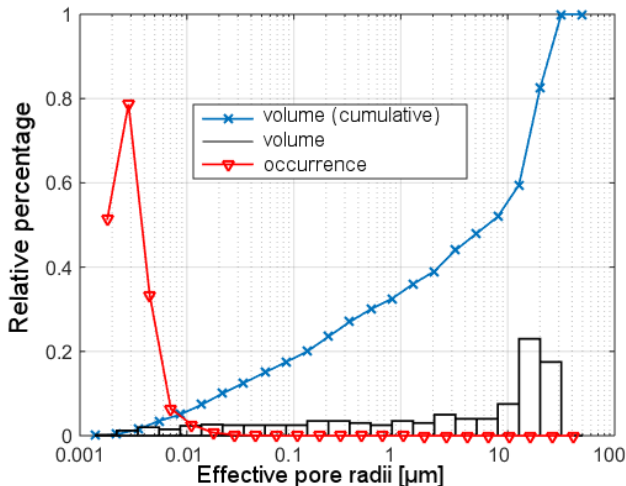
With L-r relation $L_i = 50 \cdot r_i$.

Real pore distribution

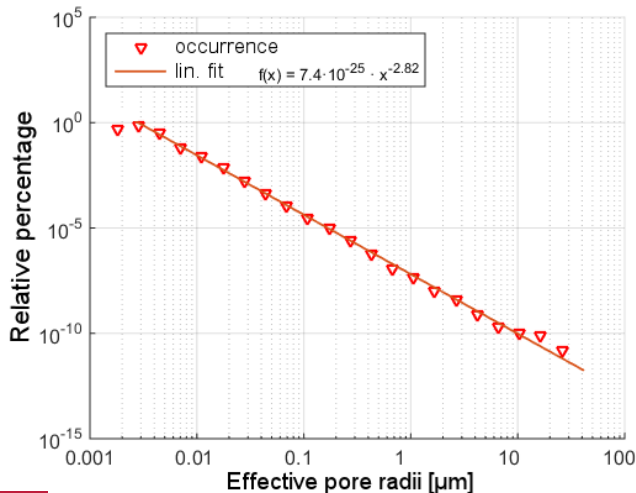


(from Weller et. al., 2011)

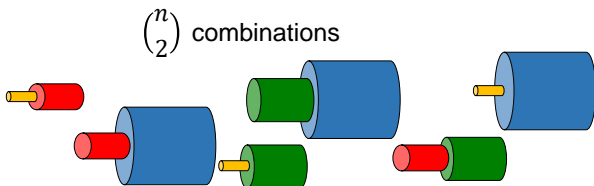
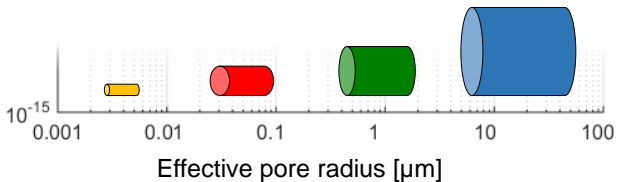
Real pore distribution



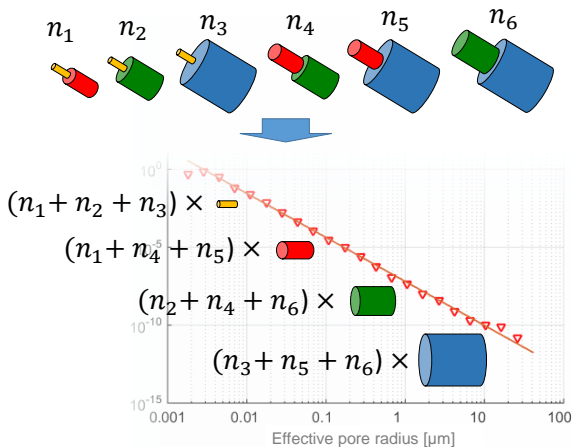
Real pore distribution



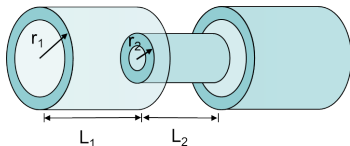
Pore combinations



Pore combinations

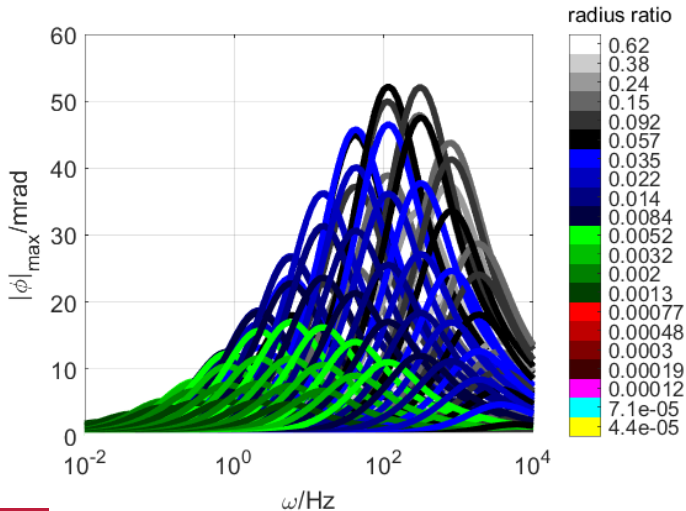


Choosing geometric parameters



- $\sigma'' - S_{por}$ - relation \rightarrow **Pore lengths** $L_i = 50 \cdot r_i$
- Real pore radii distribution \rightarrow **Pore radii combinations**

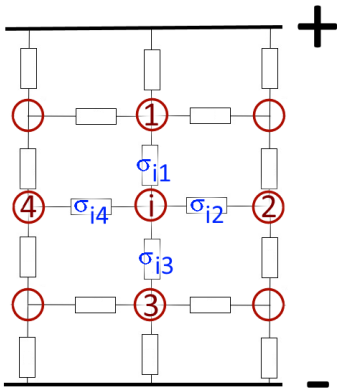
Spectra used for network



Networks - Total impedance

Finite difference method (FDM)

- potential electrodes



Networks - Total impedance

Finite difference method (FDM)

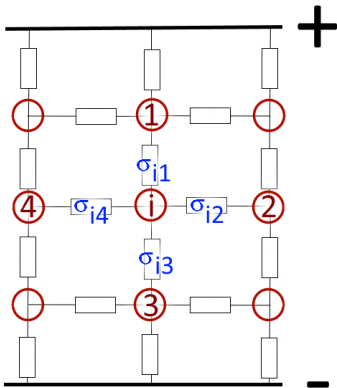
- potential electrodes
- Kirchhoff's circuit law:
conductivities $\sigma = 1/\rho$

$$\sum_j \sigma_{ij} (V_i - V_j) = 0 \quad \longrightarrow \quad \mathbf{D} \cdot \mathbf{V} = \mathbf{B}$$

D matrix from pore conductivities

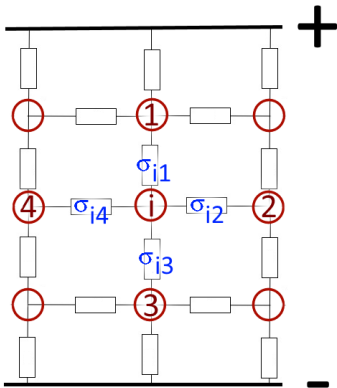
B vector from edge nodes potentials

V vector from inner nodes potential drops



Networks - Total impedance

Finite difference method (FDM)



- potential electrodes
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conductivities $\sigma = 1/\rho$

$$\sum_j \sigma_{ij} (V_i - V_j) = 0 \quad \longrightarrow \quad \mathbf{D} \cdot \mathbf{V} = \mathbf{B}$$

D matrix from pore conductivities

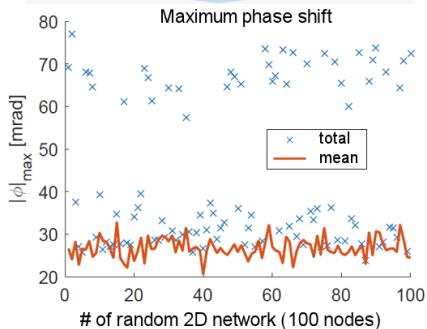
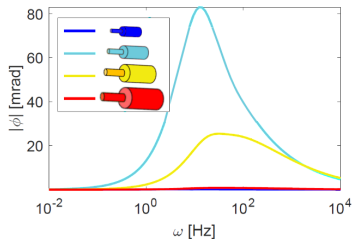
B vector from edge nodes potentials

V vector from inner nodes potential drops

- $\mathbf{V} \Rightarrow$ total impedance from edge pores currents

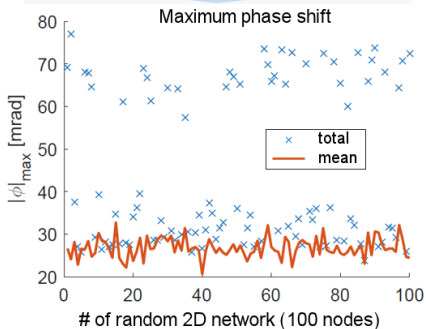
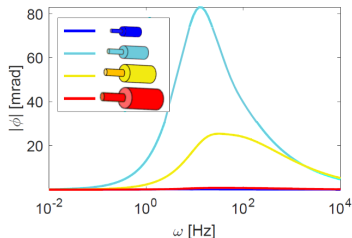
Results | Small networks

equally distrib. pore combinations

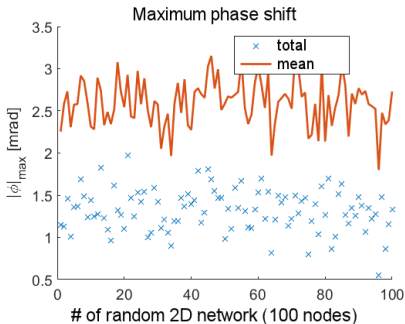
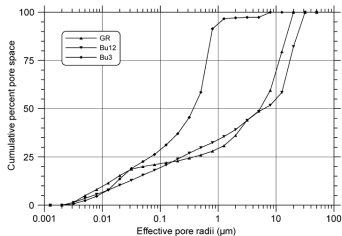


Results | Small networks

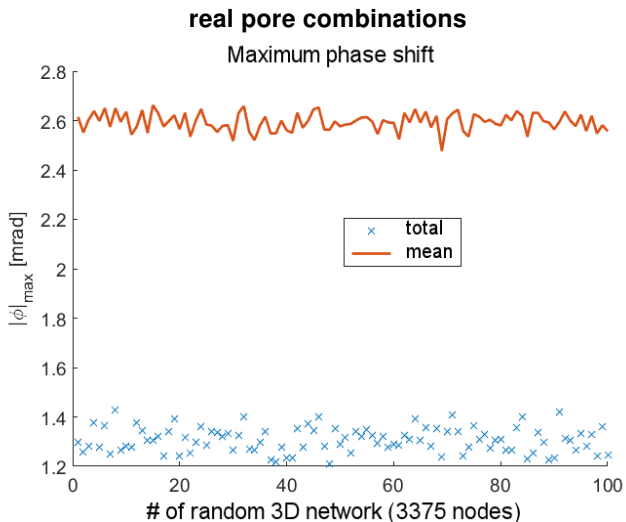
equally distrib. pore combinations



real pore distribution

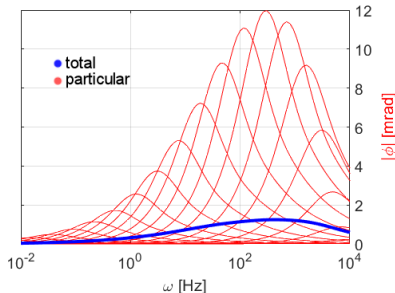


Results | Big networks



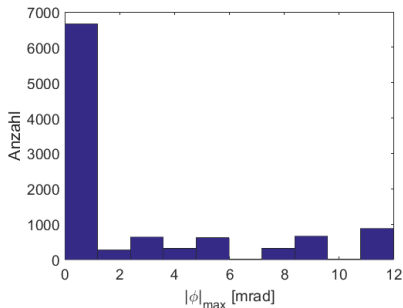
Results | Big networks

phase shift



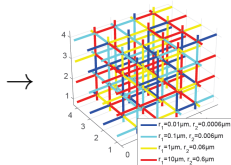
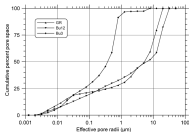
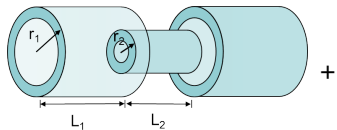
- 276 various impedances
- mean: 2.6 mrad
- network total: 1.3 mrad

maximum phase shift distribution



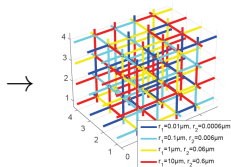
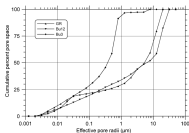
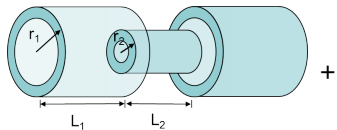
- 3375 nodes
- 10350 edges (impedances)
- biggest bin: [0 - 1.4] mrad

Conclusions



- SIP-Model: Combination of big and small pore
- Network of impedance components \rightarrow FDM \rightarrow total impedance
- Network matches real radii distribution and σ'' vs S_{por} relation

Conclusions



- SIP-Model: Combination of big and small pore
- Network of impedance components \rightarrow FDM \rightarrow total impedance
- Network matches real radii distribution and σ'' vs S_{por} relation
- Total impedance does not match the mean impedance
- Real pore distribution with pore length = $50 \times$ pore radius
 \rightarrow Realistic SIP spectra possible

Outlook

- Choosing the porelengths
- Characteristic time τ
- Constraints for possible pore combinations

Thank you for your attention!