GEOFYSIKSAMARBEJDET

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SPATIALLY CONSTRAINED INVERSION OF SKYTEM DATA
CONCEPT AND EXAMPLES

JANUARY 2008

CONTENT

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ABSTRACT

This report describes the Spatially Constrained Inversion (SCI) methodology, its main benefits, and presents an example of its applications.

It is the result of the research carried out at the HydroGeophysics Group, at the Department of Earth Sciences, University of Aarhus. The report is meant to reach a wide and varied audience, and was therefore intentionally made not too technical. Those interested in the details are referred to expanded paper about to be published in an international Journal.

The SCI is a robust methodology for quasi-3D modeling of geoelectrical and EM data (e.g., SkyTEM) of varying spatial density, using a 1D forward solution. Information migrate horizontally through spatial constraints applied between nearest neighboring soundings, and allow resolution of layers that would be locally poorly resolved. The constraints are built using the Delaunay triangulation, which ensures automatic adaptation to data density variations. In this study the SCI was applied to SkyTEM soundings, but it can be implemented also with other data types. The field case study proves that the SCI produces laterally smooth results that respect the 3D geological variations of sedimentary settings, while suppressing the elongated artifacts commonly seen in profile oriented data sets.

GeoFysikSamarbejdet 2008, Andrea Viezzoli

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INTRODUCTION

The aim of the SCI is to produce reliable quasi-3D geophysical and geological modeling from large amounts of electromagnetic data, within realistic computing times. A standard airborne electromagnetic (AEM) survey, including SkyTEM contains in excess of 500 line-km. Because full 3D inversion of such large datasets is, at moment being, unrealistic, such datasets are usually inverted using a 1D forward model. The 1D model assumption is legitimate in quasi layered sedimentary areas where it produces results that are only slightly distorted by 2D or 3D effects, as shown in published literature. There are different inversion strategies within the 1D forward assumption. For example, the output models can be simply stitched together. However, this often results in abrupt variations in neighboring models, because of noisy data and equivalent models. Models with smooth lateral variations, typical of the sedimentary areas, can be achieved by constraining adjacent output models during the inversions. A popular example is the laterally constrained inversion (LCI). The LCI is profile-oriented, in the sense that it aims at producing a continuum along a line. Constraints are set only along flight lines. There are no connections between neighbouring lines. Features that are perpendicular to lines do not benefit from in-line constraints or averaging, as no information is passed between adjacent lines. This means that profile oriented techniques favour structures following the flight direction. Producing area maps based on such methodologies can, in some cases, result in some lineation following the fight paths. In order to avoid these artefacts, and to produce a quasi 3-D modelling, the LCI was expanded to the Spatially Constrained Inversion (SCI), where constraints operate both along and across profiles. Figure 2.1 describes this concept graphically.

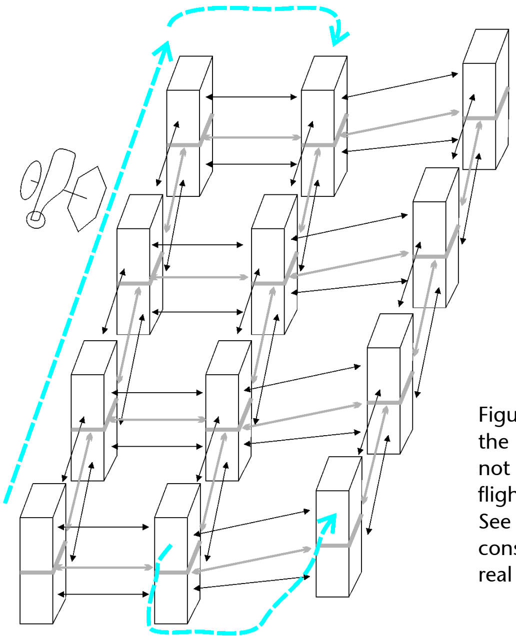

Figure 2.1 Schematic representation of the SCI concept. Constraints connect not only sounding located along the flight line, but also those across them. See figures 3.3 and 4.1 to see actual constraints between soundings from a real SkyTEM survey.

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METHODOLOGY

The mechanisms of the SCI are explained in details in a manuscript in press in the journal Geophysics (see reference list at the bottom). It is a standard least squares inversion of a layered earth, regularized through spatial constraints, which produce smooth lateral transitions. Information (including possible a priori information) migrate between neighbouring sounding, through the constraints, in all direction. Both the data and the constraints are part of the inversion, which therefore results in output models that balance between the data and the constraints. Model parameters that exert little influence on the data will be controlled by the constraints, and vice versa.

The two main issues posed by the implementation of the SCI are:

- choosing a strategy for selecting the soundings to be constrained during the inversion,
- ensuring a homogeneous flow of information throughout the entire data set.

We now expand on the first point. Soundings to be constrained need to be connected in a repeatable, not arbitrary, way. These connections also need to adapt, as much as possible, to the spatial distributions of the dataset. In our approach, we use for this purpose the so called Delaunay triangulation. See Figure 3.1 for an example of Delaunay triangulation of randomly generated points on a plane. Delaunay triangles vary in dimension according to the local data density. They adapt to the data set so that they are small and numerous in high density areas, large and fewer in low density areas. The number of connections to each sounding is not set arbitrarily, but it depends on data density and distribution. In AEM surveys, Delaunay triangulation always connects adjacent lines, which is the preliminary condition for breaking down the line orientation in the data. We decide to set the constraint between the soundings connected by Delaunay triangles, called nearest neighbors (see Figure 3.2a). This way each sounding (in this case sounding a) is linked to its "best companions",

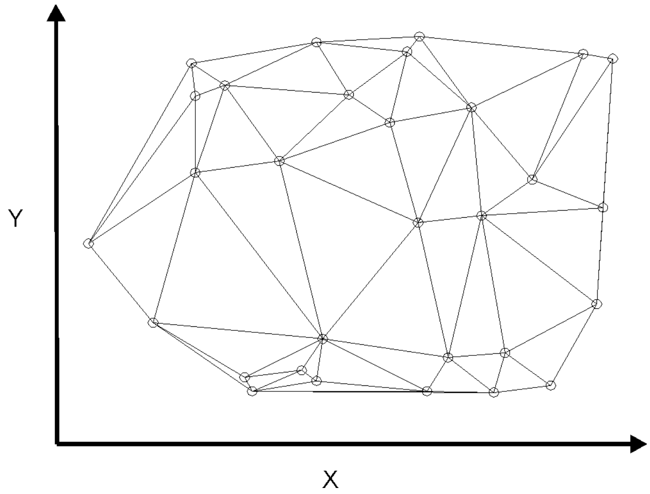

Figure 3.1 Delaunay triangulations of a randomly generated set of points on a plane.

i.e., the nearest neighbors (soundings b-g). They, in turn, are constrained to their nearest neighbors (soundings a-w, Figure 3.2b), and so on. The result is a continuum of interconnected soundings, each of which is only constrained to its nearest neighbor. Model parameter information spreads horizontally between nearest neighbors, and then to the whole data set. The strength of the constraints that result in suitable layer continuity is determined empirically, as in the LCI.

In order to perform the SCI in a CPU efficient manner, a typical data set of thousands of soundings have to be divided into smaller sub sets. Each sub set is then inverted with spatial constraints, as a unit. We produce the cells using the pre-constructed Delaunay triangles. We select a starting point (the location of a TEM sounding) randomly, and then identify its nearest neighbours, as defined above. They produce an outer border around the starting point. Then we identify the nearest neighbours to each of the points along the border. This way the cell is expanded to next order of nearest neighbours. We keep expanding the cell in a similar fashion until a predefined number of points are included in the cell. After the first cell has been built, the second one is obtained by iterative nearest neighbours expansions around one of the points along the outer border of the first cell, and so on until all data points are included in a cell. We then expand the cells around their borders, one more order of nearest neighbours. In this way we create a double overlap between neighbouring cells. This overlap region is essential for the migration of information between cells. The total number of cells is divided into as many CPUs as available, and the SCI is run in parallel. In this first run information spreads within the cells (see Figure 3.3 a). The ouput models in the overlapping regions are the weighed averages of the results of the inversions of the individual cells. The results of this first run are used as a priori information and/or starting models for a second and final SCI run. The a priori information inserted along the overlapping region brings along information coming from the neighbouring cells, which spreads back towards the centre of each cell (see Figure 3.3b).

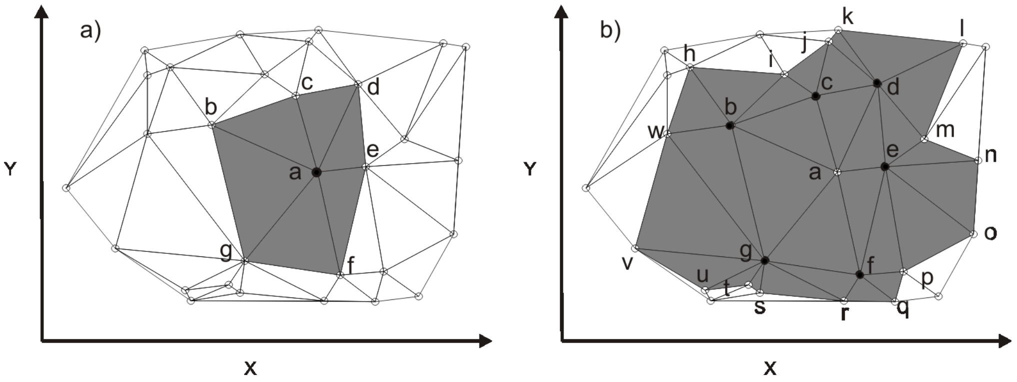

Figure 3.2 Delauany triangulation of random generated points on a plane. (a) Points **b** to **g** are nearest neighbours of point **a**. Soundings **b** to **g** are then connected to their nearest neighbors **h-w** (b), creating a continuum of interconnected soundings.

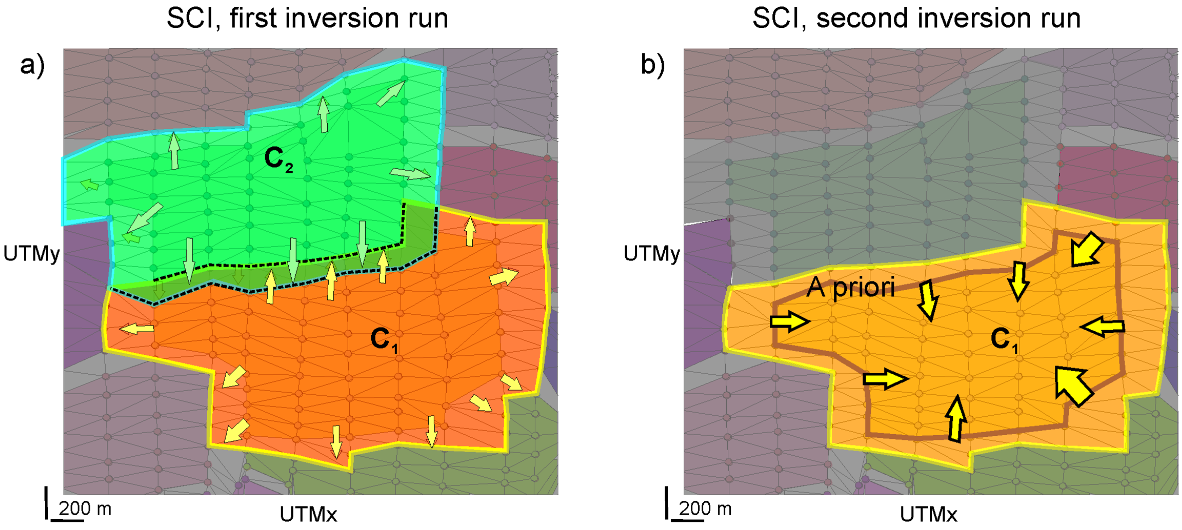

Figure 3.3 Scheme of information flow between cells in first (a) and second (b) SCI run.

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EXAMPLES

Over the past few years, the helicopter borne SkyTEM system has collected many thousands of km of TEM data, many of them in sedimentary environments for groundwater exploration. We present a case study from a 80 km^2 survey in the Stevnstrup area, in eastern Jutland, in Denmark, as seen in Figure 4.1a. Each black dot represents a sounding, which is either a low moment or a high-moment sounding. Average spacing between flight lines is 250 m. Many soundings near roads and power lines have been removed because of transmitter-induced couplings to power-lines and to cables buried along the roads. The full data set contains 5477 soundings. Of these, approximately half are high moment (~60000 Am^2), and the other half low moment (~10000 Am^2) soundings. Figure 4.1b shows the Delaunay triangulation of the SkyTEM soundings. Note how the Delaunay triangulation always connects adjacent lines. Figure 4.1c reports a frequency histogram of the number of constraints per sounding, which are about 6 in average.

In general terms, the geology of the survey area consists of Danien lime-

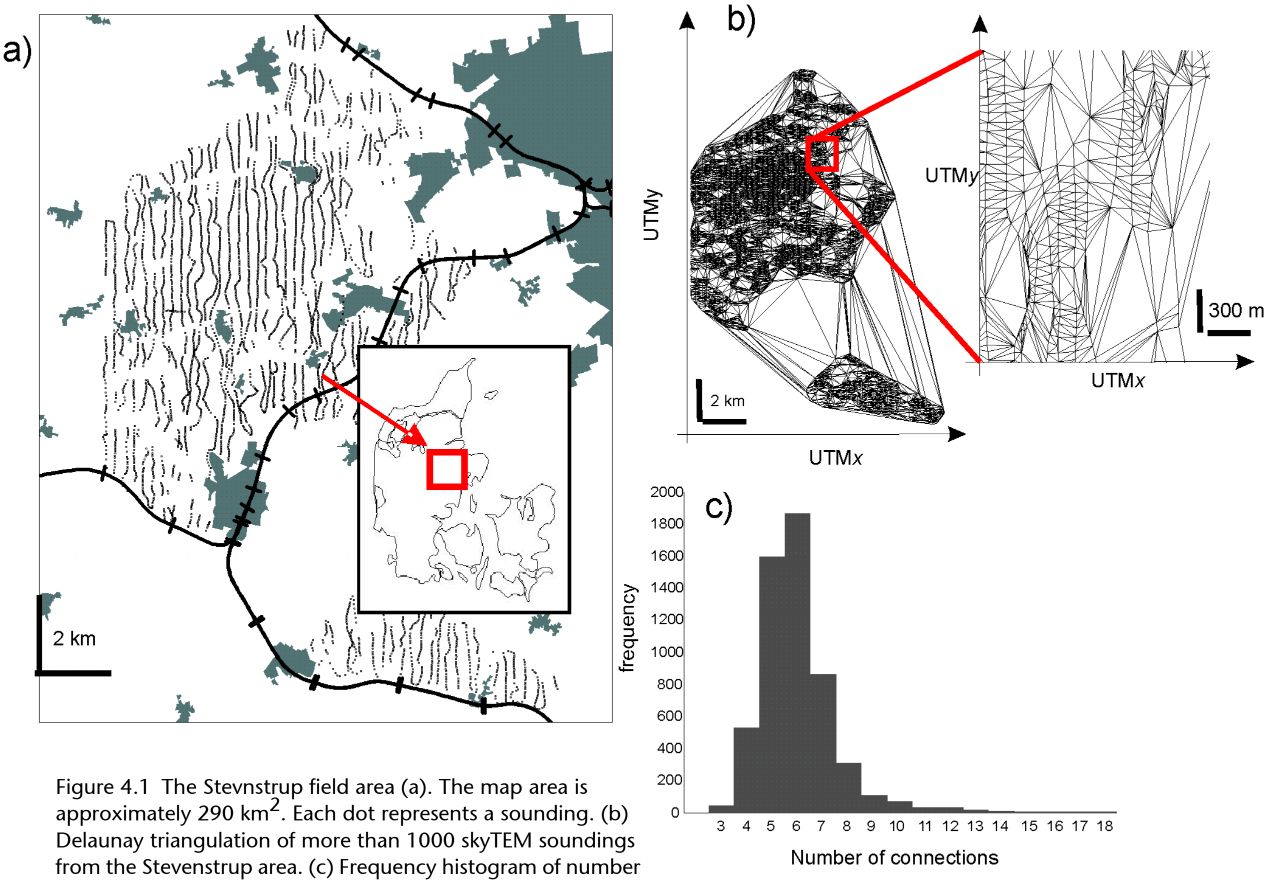

Figure 4.1 The Stevnstrup field area (a). The map area is approximately 290 km^2. Each dot represents a sounding. (b) Delaunay triangulation of more than 1000 skyTEM soundings from the Stevenstrup area. (c) Frequency histogram of number of connections between soundings in the Delaunay triangulation.

stone at the bottom. The limestone is saturated with residual saltwater (average resistivity of = 2 Ωm) in the very deep parts and infiltrating freshwater (30-100 Ωm) in the more superficial parts. On top of this is 0-100 m heavy, Paleogene clay with an average resistivity of 25 Ωm. The uppermost part of the sequence is till consisting of a varying clay mineral content and glacial sands. It was expected that one or more buried valleys were incised into the Paleogene clay. These valleys are filled with outwash sand and gravel (50-80 Ωm), and represent important aquifer structures.

We compare the results of three different inversion approaches: a) stitched together independendent 1D inversion of individual soundings (i.e., no constraints in any direction), b) LCI, and c) SCI. Each inversion has the same starting models (in this case a uniform half space of 50 Ωm), and 5 layers.

First, we present the results in the form of mean resistivity values at different elevation intervals (Figure 4.2), and then in the form of profiles (Figure 4.3 and 4.4). Note that the colour scales in all the maps and profiles use the blue tones representing resistive structures, and red tones the conductive ones. The average resistivity maps, at both elevation intervals, clearly demonstrate the effect of the constraints. The LCI (Figures 4.2c and 4.2d) promotes along-line continuity, with respect to the stitched together single-site inversion (Figures 4.2 a and 4.2b). For example, note the area that delimits a buried valley, which runs in the east-west direction (delimited by the purple rectangle in Figure 4.2c). However, the LCI also introduces elongated features coincident to the flight lines (more evident in the areas delimited by the black rectangles in figures 4.2c and 4.2d). The SCI (Figures 4.2e and 4.2f), on the contrary, produces smooth variations in every direction. It clearly delineates the borders of the buried valley. It resolves continuous west-east features, especially noticeable in the region of the valley, that are not so well identified by the other two methods. The good conductor present at depth in the valley (Figure 4.2e) represents residual saline water in a limestone host, whereas the shallower resistive structure visible in Figure 4.2f corresponds to unconsolidated sediments. Figure4.3 d shows a sketch of the vertical section of the geology of the area.

The complete SCI was carried out on the data set more than once, with different starting points for the creation of cells, which therefore had different locations, shapes and sizes. Results, not shown here, proved that the SCI is robust with respect to the choice of the starting point, i.e., of the cells geometry.

Figures 4.3 and 4.4 show the cross sections of the two profiles drawn onto the maps in Figure 4.2. The two profiles allow comparison of the results of the inversion methodologies along different directions. In both, the single site stitched together inversion gives the least lateral continuity, as expected. The south-north profile in Figure 4.3 follows a flight line, and therefore also the chain of soundings constrained in the LCI. This should therefore produce good results, apart from possible minor distortions due to 2D effects along the edges of the buried valley. A sketch of the geological cross section inferred from available geological models is given in Figure 4.3d. Both the LCI and the SCI identify correctly all the main geological units. The single site inversion fails to delineate the boundary between clay and limestone recorded in the proximity of the borehole, although it does define the boundary in other areas of the profile. The minor difference between SCI and LCI is in the detection of the whole clay-limestone boundary in the northern portion of the profile, which the SCI defines more continuously. This result is due to the fact that the constraints set in

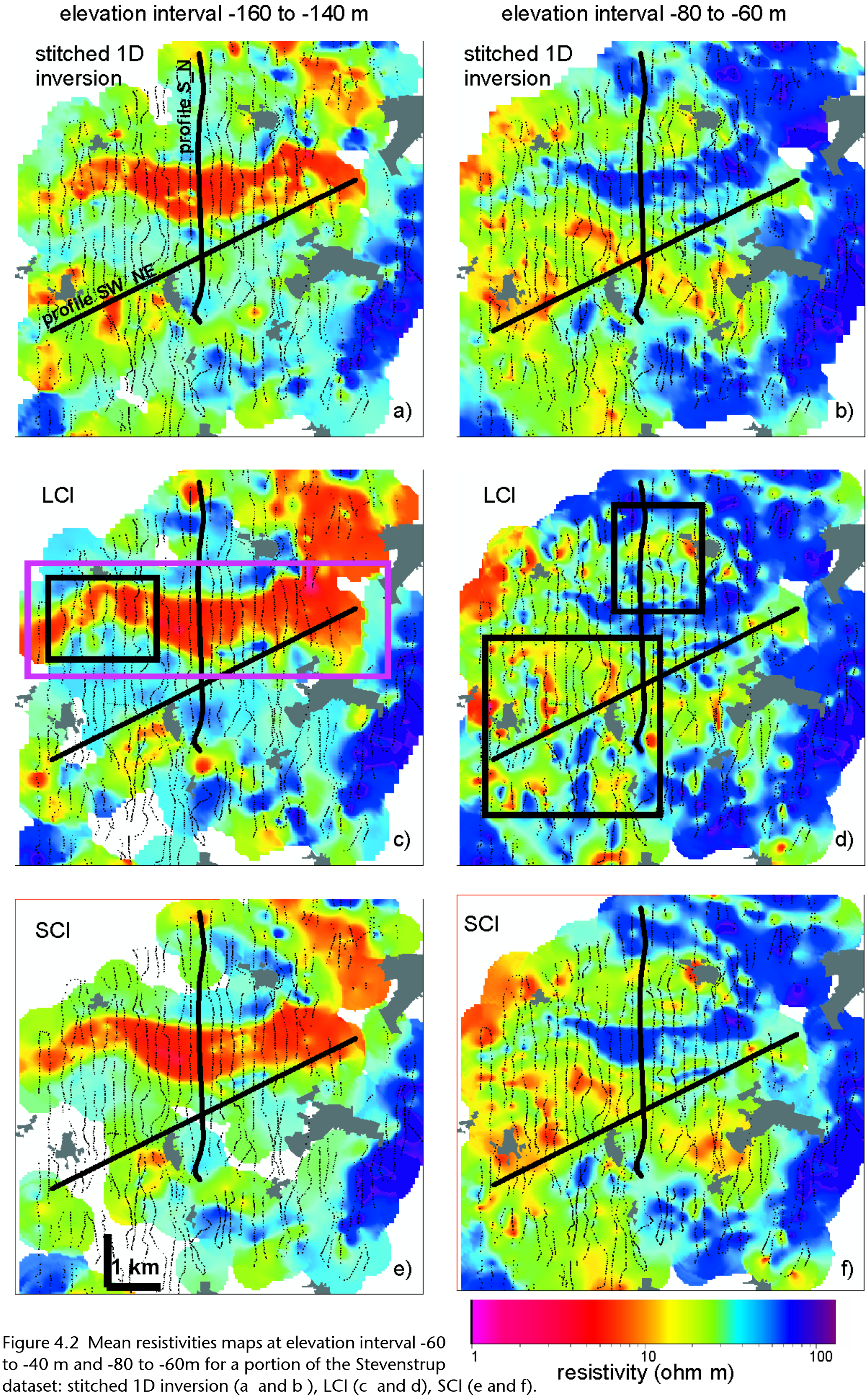

Figure 4.2 Mean resistivities maps at elevation interval -60 to -40 m and -80 to -60m for a portion of the Stevenstrup dataset: stitched 1D inversion (a and b), LCI (c and d), SCI (e and f). Black dots represent SkyTEM soundings, black lines the profiles shown in Figures 4.3 and 4.4. The purple rectangle shows the area of the buried valley. Please note the colour scale, with red being conductive and blue resistive.

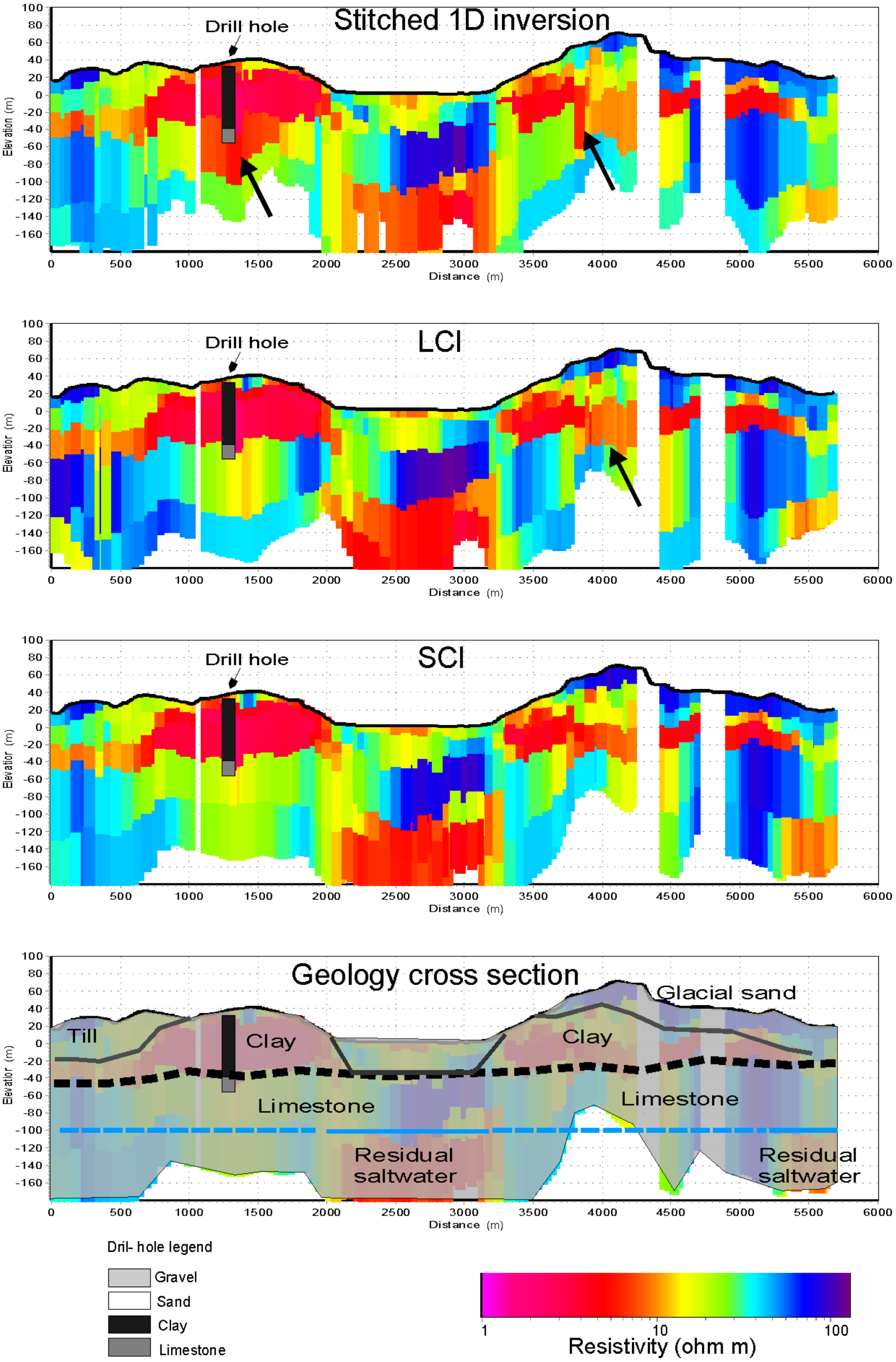

Figure 4.3 Resistivity cross section for profile SW_NE: (a) single site inversion stitched together, (b) LCI, (c) SCI (d) sketch of geology cross section.

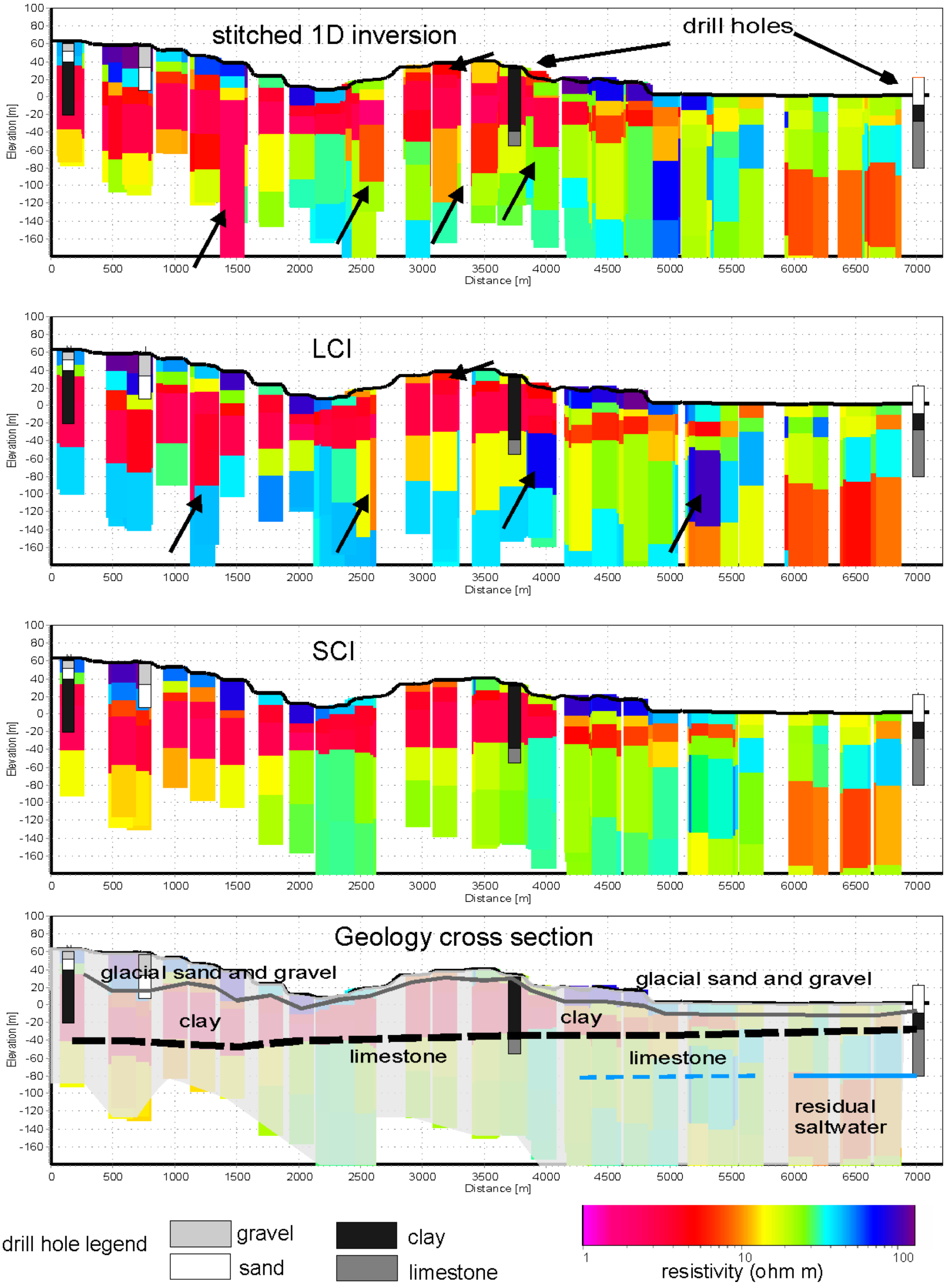

Figure 4.4 Resistivity cross section for profile SW_NE: (a) single site inversion stitched together, (b) LCI, (c) SCI (d) sketch of geology cross section.

the SCI allow model parameter information to migrate also across the flight path, not only along it (as in the LCI). Model parameters are, therefore, better resolved in the SCI. In Figure 4.3, black bold arrows indicate the location of the main discrepancies between the results of the SCI and of the other methods. In the portions of the profile where the limestone is overlain by a thick clay cover, its absolute resistivity values are underestimated. The thick clay layer also masks the presence of deeper residual saltwater.

Figure 4.4 displays less continuity because the direction of the profile SW-NE (see Figure 4.1 for location of profile) does not coincide with flight lines, and thus has a lower sounding density. Both the SCI and the LCI results agree substantially with the available geological model (Figure 4.4d). The SCI, however, provides more continuous results overall, both at the boundary between the shallow resistive layers of glacial sediments and clay, and at depth, along the clay limestone boundary. Black bold arrows indicate once again the main differences between the SCI and of the other two methods.

The SCI reveals the same overall geological structures as LCI but that they are significantly different in detail. SCI recovers the actual geology of the area better and the pictures are much more coherent compared to those based on profile-oriented LCI, and the individual soundings inversions. We also analyzed the model parameter covariance and the data-fit residual, in order to give a mathematical evaluation of the results produced. Such analysis proved that the SCI decreases the uncertainty of model parameters, as expected, while still fitting the data.

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MAIN BENEFITS OF THE SCI

The main benefits of the SCI are:

- It produces quasi 3D modeling of EM data, using a 1D forward approximation and 3D spatial constraints, in realistic computing times.
- Having 3D constraints, it makes the most of the inherent spatial coherency of the data. Therefore, in sedimentary environments, the results are spatially smooth.
- It prevents along flight-line artifacts.
- It is a parallel procedure that allows to spread the load of the inversion on as many CPUs as are available. This implies that it can handle arbitrarily large datasets.
- It adapts automatically to data density variations. It can therefore be applied to AEM, ground-based data, or a combination of the two.
- The spatial constraints allow resolution of model parameters that would be locally poorly resolved.

Like the LCI, the SCI also

- produces model parameter sensitivity analysis, and maps of data fit.
- allows a priori information (e.g., from borehole data) to be inserted from any location within the survey area.

The SCI is slightly slower than the LCI. Apart from this, there are no real drawbacks in the application of the SCI, with respect to the LCI, or to single site inversions. Our experience is that, provided the constraints are not set too tight, both the SCI and the LCI always produce better results , both graphically and numerically, than the single site inversion. In cases of extremely good data quality the constraints have smaller influence on the output models, and the SCI results may not differ significantly from the LCI results. Another instance when the SCI would not give major benefits over the LCI is in datasets where soundings do not cover an area, but are all located along a single line (e.g., flying above a river, or a test line, or in MEP measurements carried out along a single profile). In such cases it may be easier to set up an LCI only.

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CONCLUSIONS

The SCI applies horizontal constraints for ensuring lateral continuity, improving resolution of model parameters for single stations that are not well resolved by the data from that station alone. Use of Delaunay triangulation for the constraints allows the SCI to adapt efficiently to data density variations. In profile-oriented data sets, it ensures a connection between adjacent lines by means of across-line constraints. Therefore, it eliminates the common elongated features that often coincide with the direction of the survey (e.g., flight lines), and that distort the continuity of geological units across lines. Although based on a 1D forward model, the SCI results in a computationally practical, quasi 3D inversion of EM data.

The SCI can be applied to different data types. In the study presented here the SCI was successfully applied to quasi 3D modeling of TEM data in a sedimentary environment. The mean resistivity slice map, the profiles, the sensitivity analysis of the model parameters, and the analysis of the data-fit prove that, overall, the SCI, while fitting the data, produces laterally smooth, well-determined output models that resemble the known geology of the area better than stitched together original inversions, and also better than a profile-oriented inversion methodology, like the LCI. The SCI allowed a significant improvement in the mapping of the intermediate clay-limestone interface.

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