Predictive relationships for the permeability of unconsolidated sands based on SIP and pore surface fractal dimensions

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Permeability prediction based on S_{por} and D

- For sandstone samples Zhang &Weller (2014) have demonstrated a relationship between S_{por} and the fractal dimension (D) of the pore surface and incorporated D into a more general form of the PaRiS model (originally proposed by Pape et al. (1987)) to predict permeability

$$k = \frac{1}{8F}(\lambda_{N_2})^{\frac{2D-4}{D-3}}\left(\frac{S_{por}}{2}\right)^{\frac{2}{D-3}} \quad (1)$$

- We (Joseph et al., 2016) have made SIP and permeability measurements on unconsolidated sand samples both of individual size fractions and mixtures of sizes.
- For these samples S_{por} has been calculated from measurements of porosity and the masses and mean grain diameters of the samples.
- From the calculated values of S_{por} the fractal dimension of the pore surface has been estimated using a relationship, based on that of Zhang & Weller (2014)

$$D = 2 + \frac{\log(S_{por}) - \log(2/r_{eff})}{\log(r_{eff}/\lambda_{H_2O})} \quad (2)$$

in which the effective hydraulic radius $r_{eff} = \sqrt{8kF}$.

- Compared to sandstone samples reported by Zhang & Weller, for which D increases with S_{por}, we find D for our unconsolidated samples to be very close to 2 for $r_{eff} \geq 10$ µm (Figure 1).
- A plot of permeability predicted by (1) against measured permeability is in excellent agreement (Figure 2).
- Use of average values of D (2.0 for unconsolidated samples and 2.307 for sandstone samples) still gives good predictions of permeability (Figure 2).

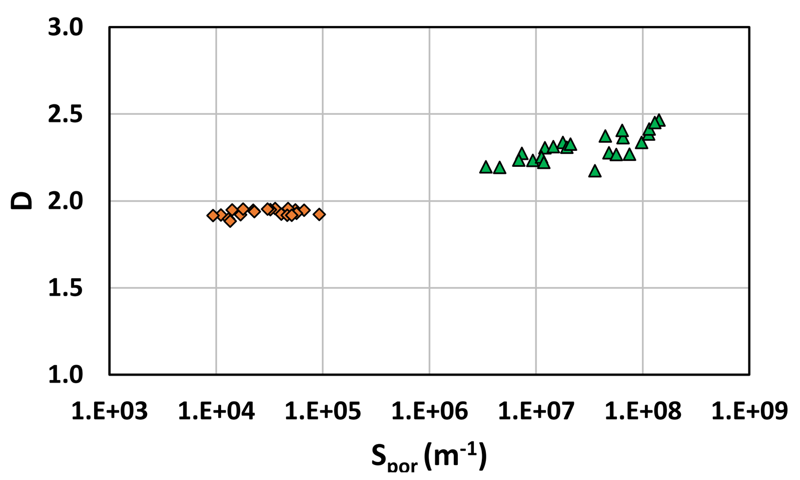

Figure 1: Calculated value of pore fractal dimension D as a function of specific internal surface S_{por}. Diamonds– unconsolidated sand samples; triangles– sandstone samples (Zhang & Weller, 2014).

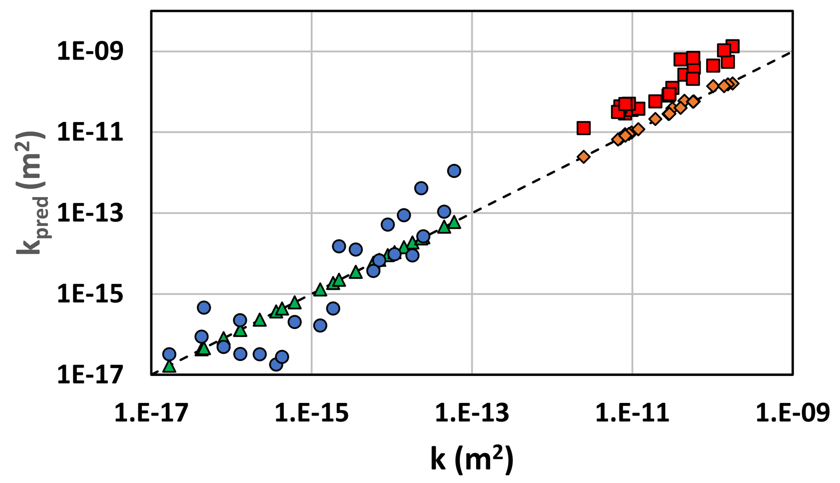

Figure 2: Permeability predicted by equation (1) plotted against measured permeability. Diamonds – unconsolidated sand samples using calculated values of D; squares - unconsolidated sand samples using D = 2.0; triangles – sandstone samples (Zhang & Weller, 2014) using measured values of D; circles - . sandstone samples using D = 2.307.

Relationships between S_{por} and SIP parameters

- Although the use of the PaRiS model gives excellent predictions of permeability, the requirement to have a knowledge of S_{por} makes it difficult to apply in a field setting. It is therefore appropriate to seek relationships between S_{por} and parameters that are readily obtainable from field measurements.
- Two such parameters are the imaginary part of the complex conductivity (σ'') measured at a frequency of 1 Hz, and the Cole-Cole time constant (τ).
- Derived fits between these parameters and S_{por} for the unconsolidated sands are shown in Figures 3 and 4 and, with the assumption of D = 2, lead to predictive relationships for permeability based on σ'' and τ.

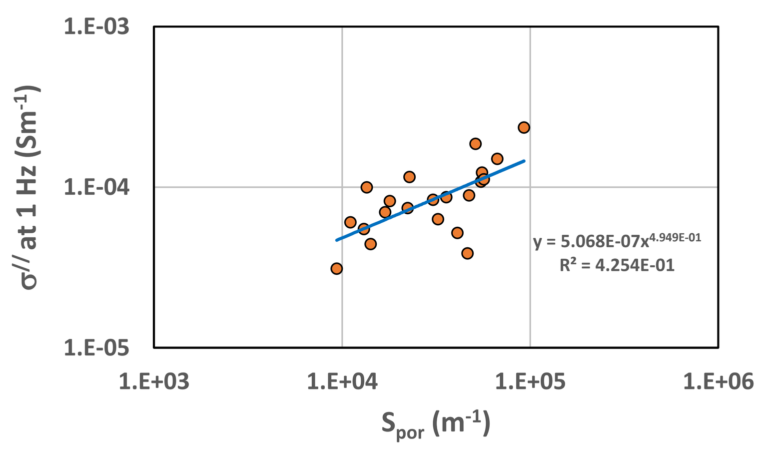

Figure 3: Relationship between σ''_{1Hz}, the imaginary part of the conductivity at a frequency of 1 Hz, and S_{por}.

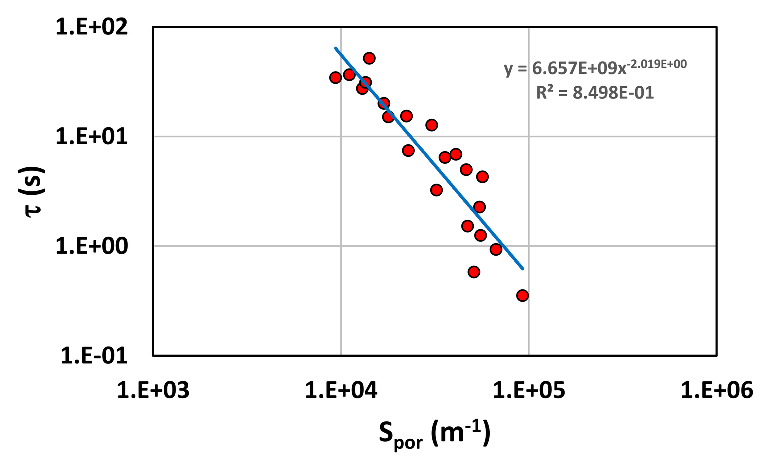

Figure 4: Relationship between τ, the Cole-Cole time constant, and S_{por}.

Permeability prediction based on σ'' and τ

- Assuming D = 2 for the unconsolidated sands, the predictive relationship for permeability based on the value of σ'' at 1 Hz is:

$$k = \frac{1.357 \times 10^{-28}}{8F}\sigma''^{-4.739} \quad (3)$$

with k in m^2 and σ'' in S/m.

- The comparable relationship for predicting permeability from the Cole-Cole time constant is

$$k = \frac{5.247 \times 10^{-10}}{8F}\tau^{1.113} \quad (4)$$

with k in m^2 and τ in seconds.

- Plots of the permeability predicted by these equations against the measured permeability are shown in Figures 5 and 6.
- For these samples, predictions based on σ'' show a considerable degree of scatter (Figure 5).
- Predictions based on τ (Figure 6(a)) are much less scattered but tend to overestimate the permeability.
- If an average value of D (1.932) is used, and equations (3) and (4) adjusted appropriately, much better predictions of k are obtained (e.g. Figure 6(b)).

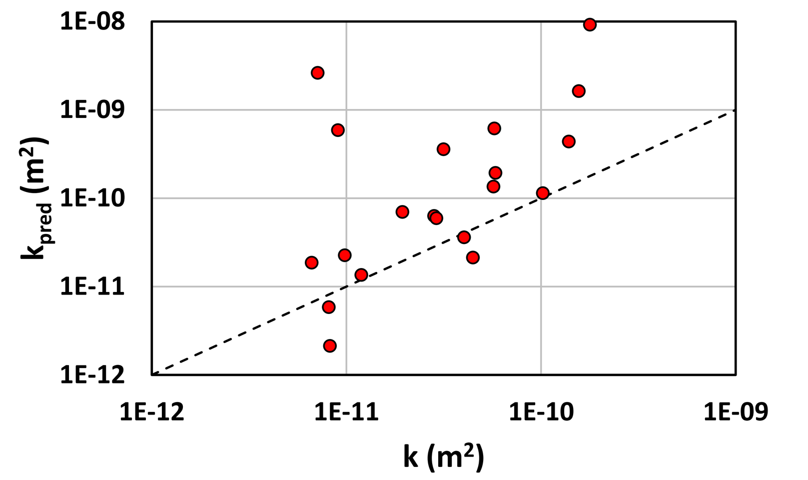

Figure 5: Plot of permeability predicted by equation (3) against measured permeability for unconsolidated sand samples.

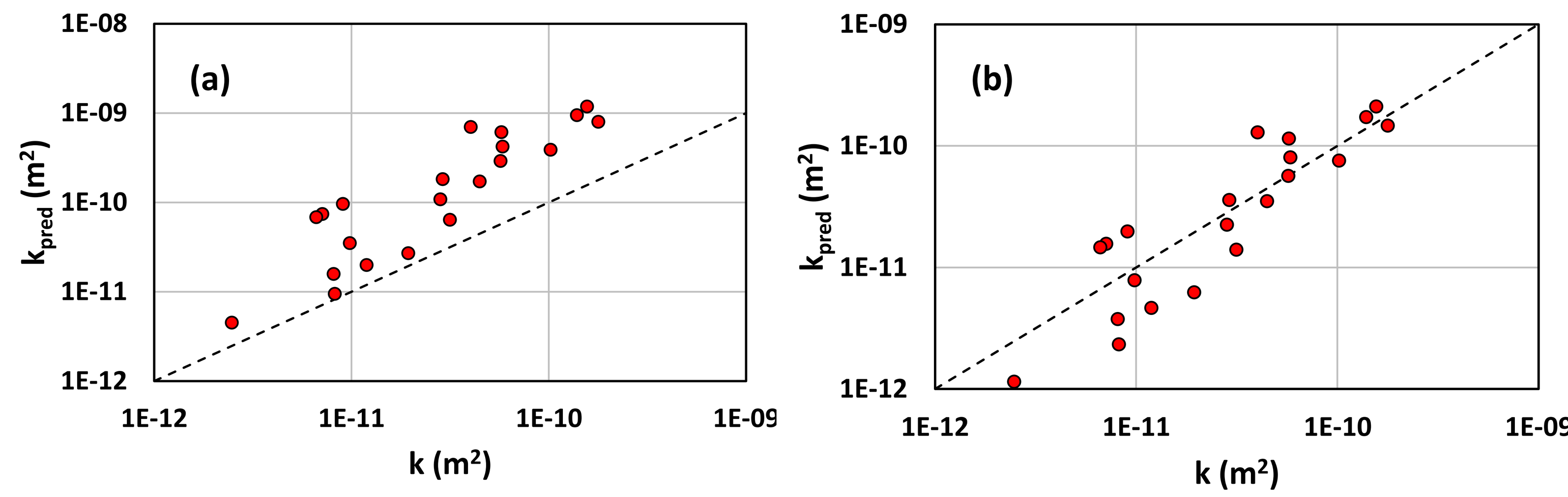

Figure 6: (a) Plot of permeability predicted by equation (4) against measured permeability for unconsolidated sand samples; (b) improved prediction of permeability based on the Cole-Cole time constant obtained by using an average value of pore fractal dimension (D) of 1.932.

Conclusions

- ✓ For unconsolidated sand samples for which the effective hydraulic radius is greater than 10 µm the calculated pore surface fractal dimension is close to, but just less than, 2.
- ✓ Using the calculated values of D in the generalized PaRiS model (equation (1)) gives excellent predictions of permeability. Using a constant value of 2 leads to a slight overestimate of permeability.
- ✓ Power law relationships between both σ'' and τ and S_{por} allow predictive relationships based on these parameters to be developed.
- ✓ Assumption of D = 2 in these relationships, as representative of unconsolidated samples, leads to an overestimation of k. However, use of an average value of D, slightly lower than 2, gives improved predictions using both σ'' and τ.

References

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