Comparison of Cole-Cole and Constant Phase Angle modeling in Time-Domain Induced Polarization

M. LAJAUNIE, P. K. MAURYA AND G. FIANDACA



Content

- 1. Analytical descriptions of the induced polarization
 - Cole-Cole model
 - Constant Phase Angle (CPA) model
 - Ambiguity in time domain
- 2. Synthetic tests
- 3. Comparison with field data



Two classes of Induced Polarization signal



Two classes of Induced Polarization signal



Phase angle changing with frequency •

Cole-Cole model

Phase angle almost constant over the • frequency range investigated

Constant phase angle (CPA) model

Formula :

$$\zeta_{Cole-Cole} = \rho \left(1 - m_0 \left(1 - \frac{1}{1 + (i\omega\tau)^C} \right) \right)$$

Time domain :





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Cole-Cole model $\rho = 100 \ \Omega.m$ m0 = 40 mV/V $\tau = 1$ c = 0.5

Formula :

$$\zeta_{Cole-Cole} = \rho \left(1 - m_0 \left(1 - \frac{1}{1 + (i\omega\tau)^C} \right) \right)$$

Time domain :





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c = 0.3

Formula :

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Time domain :





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Cole-Cole model $\rho = 100 \ \Omega.m$ m0 = 40 mV/V $\tau = 1$ c = 0.5 c = 0.3 $\tau = 0.1$

Formula :

$$\zeta_{Cole-Cole} = \rho \left(1 - m_0 \left(1 - \frac{1}{1 + (i\omega\tau)^C} \right) \right)$$

Time domain :





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 Cole-Cole model $\rho = 100 \ \Omega.m$ m0 = 40 mV/V $\tau = 1$ c = 0.5
 c = 0.3
 $\tau = 0.1$
 m0 = 10

Constant Phase Angle (CPA) model

Formula :

$$\zeta_{CPA} = K(i\omega)^{-b}$$

Time domain :





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CPA model ϕ = 6.64 mrad

Constant Phase Angle (CPA) model

Formula :

$$\zeta_{CPA} = K(i\omega)^{-b}$$

Time domain :





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CPA model ϕ = 6.64 mrad

 ϕ = 4.53 mrad

Constant Phase Angle (CPA) model

Formula :

 $\zeta_{Drake} = K(i\omega + \omega_L)^{-b}$

Time domain :





Title

COLE-COLE MODEL



CPA MODEL

- Phase [miliradians]



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Title

COLE-COLE MODEL



CPA MODEL



Can we distinguish between the two classes with time domain data?

COLE-COLE MODEL

CPA MODEL





Content

- 1. Analytical descriptions of the induced polarization
 - Cole-Cole model
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Content

1. Analytical descriptions of the induced

polarization

- Cole-Cole model
- Constant Phase Angle (CPA) model
- Ambiguity in time domain

2. Synthetic tests

3. Comparison with field data



Parameters for the models

ρ	m0	τ	С
100 ohm.m	40 mV/V	[0.001 - 10]	[0.1; 0.3; 0.5]

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Parameters for the models

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Frequency exponent c



Small c values increase the similarity between the two models





















The role of τ depends highly on the acquisition time range



Small c values increase the similarity between the two models













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Two decades of data is a minimum to distinguish both models



The shape of the CPA decays is ENTIRELY determined by the acquisition settings (current On-time and Off-time, stack...)

The role of $\boldsymbol{\tau}$ depends highly on the acquisition time range



Small c values increase the similarity between the two models







Content

1. Analytical descriptions of the induced

polarization

- Cole-Cole model
- Constant Phase Angle (CPA) model
- Ambiguity in time domain
- 2. Synthetic tests
- 3. Comparison with field data



Field data – 2D comparison



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- 2D variations give complex shapes
- Cole-Cole : able to retrieve the complex shapes
- CPA : unable to retrieve the complex shapes



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-> no variation of the shape of IP decays also in 2D



What about the phase ?





The distinctness depends highly on the acquisition time range :

	High c	Low c
More than 2 decades of time	Highly distinguishable	Not very distinguisha
Less than 2 decades of time	Not very distinguishable	Not distinguishable

Specific values for Tau and c increase the similarity between the 2 models

low c

Tau in the center of the logarithmic time range

2D field data can contain complex shapes that only the Cole-Cole model can fit.

In any case, the inverted phase shift is in very good agreement with the order of magnitude of the maximum phase shift.



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Thank you for your attention !



