Relationship between Cole-Cole model parameters and spectral decomposition parameters derived from SIP data

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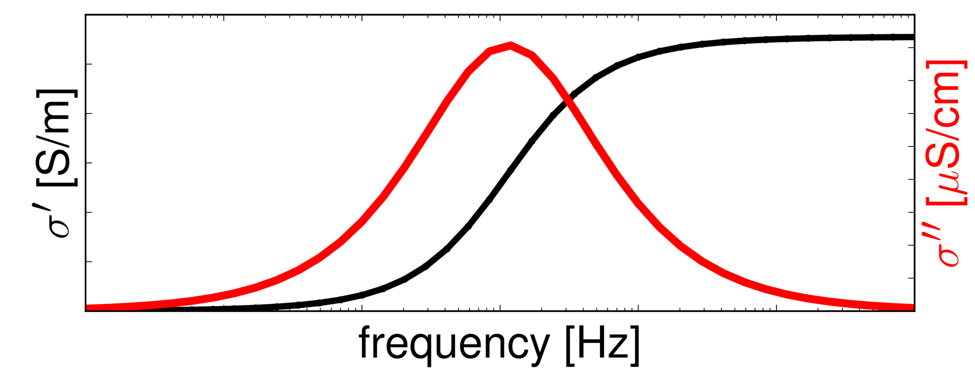
Introduction

We here present a numerical study which compares the resulting parameters of the Debye and Warburg decomposition schemes with the Cole-Cole parameters used to generate the SIP signatures. The Cole-Cole (CC) model is commonly used to analyze the characteristics of SIP data. However, decomposition approaches describing a given SIP signature by means of a superposition of a large number of polarization terms have recently been increasingly adopted. SIP parameters from different phenomenological models have been jointly used to improve data coverage of certain relationships or processes. Slight variations in model parameters can possibly lead to interpretation errors, which are investigated in this study by means of numerical experiments.

Cole-Cole model



The Cole-Cole model describes the complex resistivity ($\hat{\rho}$) (Pelton et al., 1978):

$$\hat{\rho}(\omega) = \rho_0 \left[1 - m\left(1 - \frac{1}{1 + (j\omega\tau)^c}\right)\right],$$

where ρ_0 is the DC resistivity, m the chargeability, τ the characteristic relaxation time, and c the CC exponent.

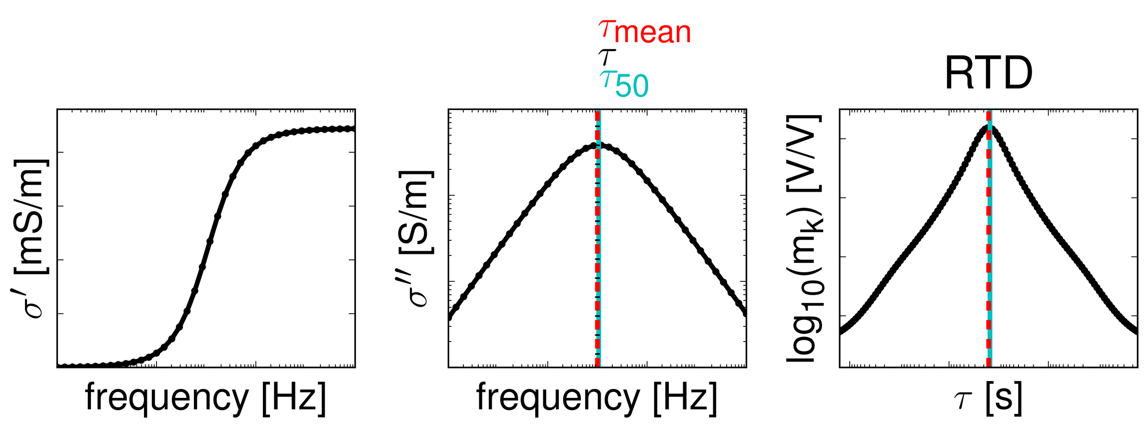
Cole-Cole decomposition

The Cole-Cole decomposition (CCD) describes $\hat{\rho}$ using a superposition of multiple relaxation terms:

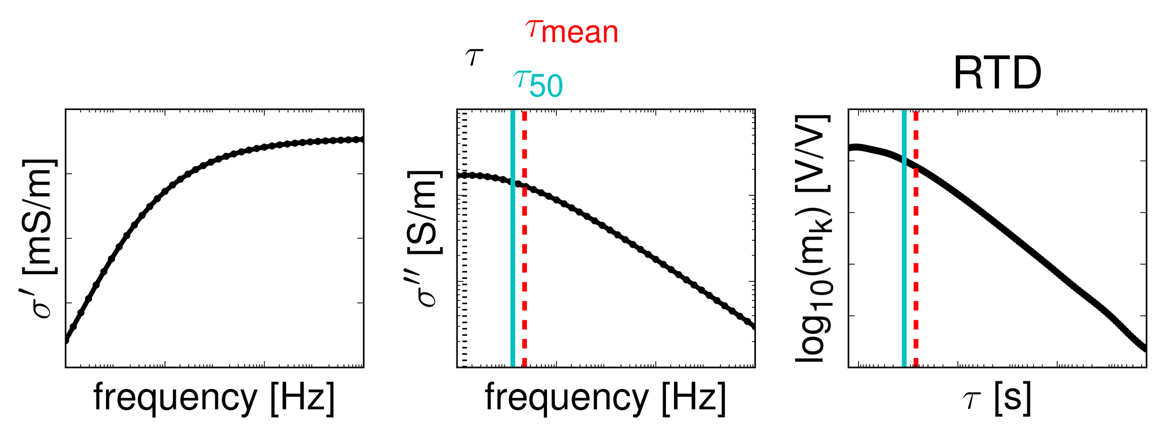
$$\hat{\rho}(\omega) = \rho_0 \left(1 - \sum_{k=1}^{N} m_k \left[1 - \frac{1}{1 + (j\omega\tau_k)^{\tilde{c}}}\right]\right),$$

where m_k is the k-th chargeability weight of the k-th relaxation time τ_k. The imaginary number is denoted by j and the frequency dispersion of each relaxation term is controlled by the fixed constant c. The distribution of weights m_k produced by the decomposition is called the relaxation time distribution (RTD). Important settings of the CCD are the number of relaxation times per frequency decade and the range of relaxation times (Weigand and Kemna, 2016a).

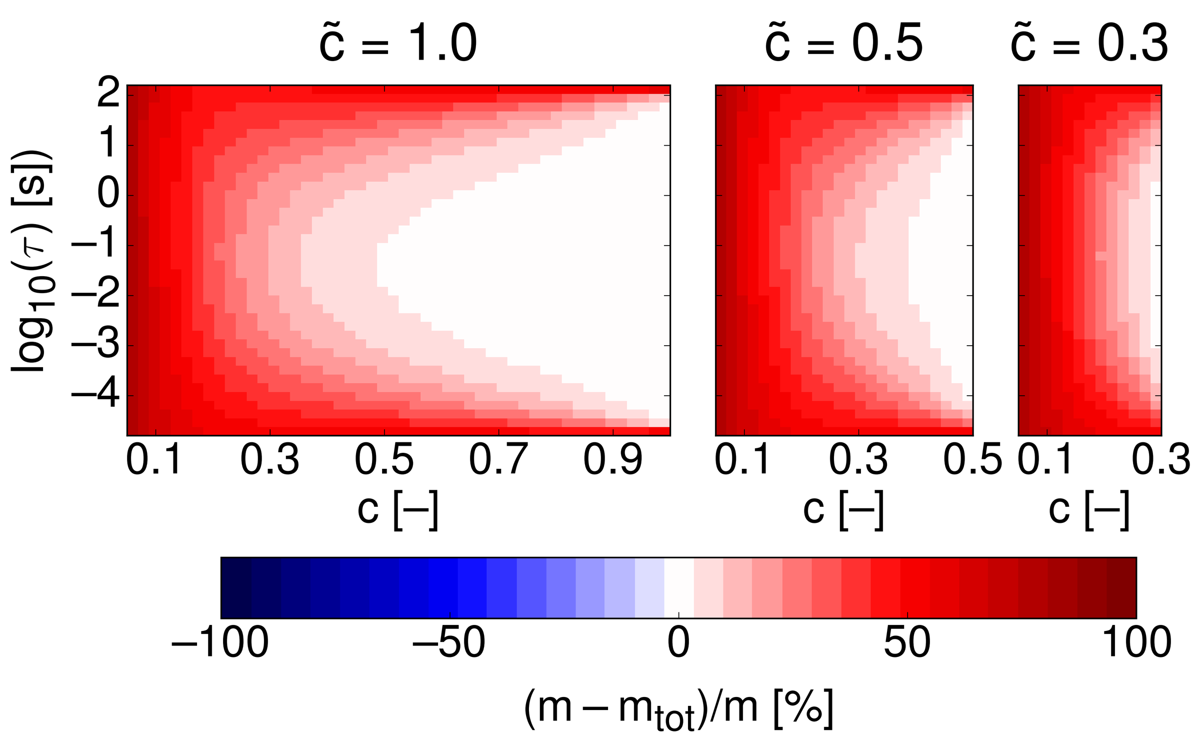
Integral parameters



- *total chargeability* $m_{\text{tot}} = \sum_{k=1}^{N} m_k$
- τ_{50} is the relaxation time at which 50 % of the total chargeability is reached.
- $\tau_{\text{mean}} = \exp\left(\frac{\sum_{k=1}^{N} m_k \log(\tau_k)}{\sum_{k=1}^{N} m_k}\right)$

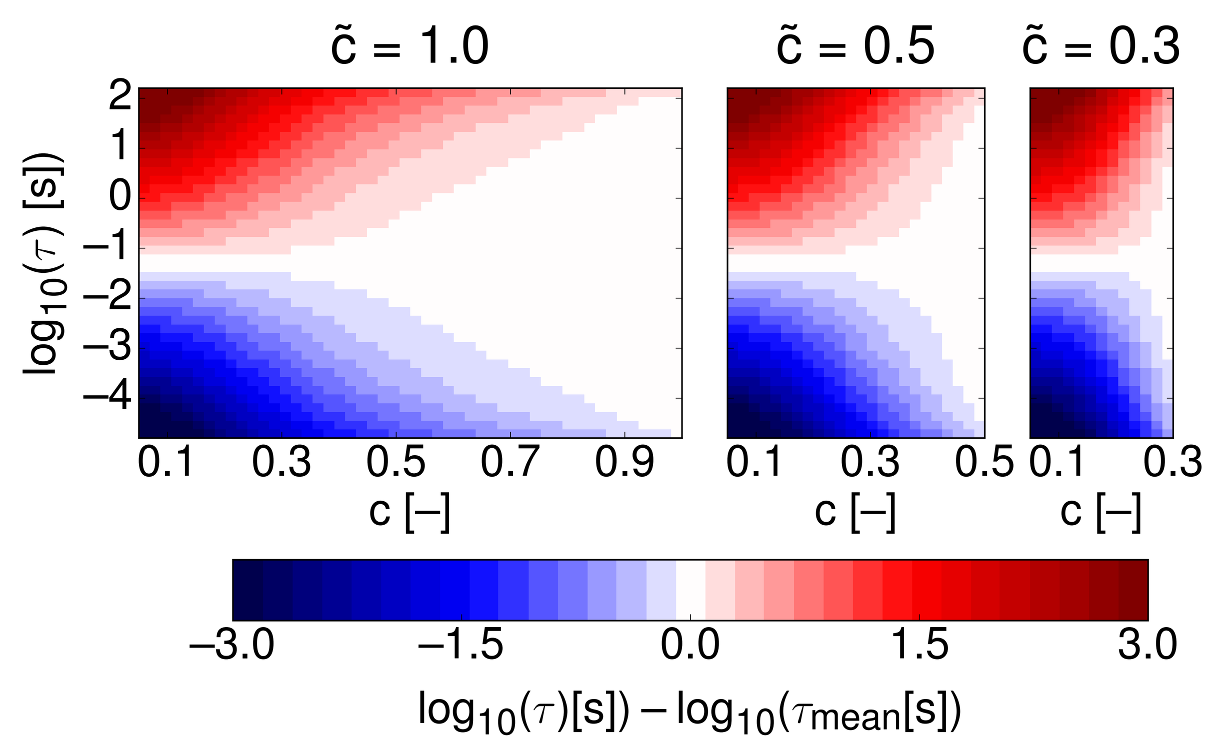


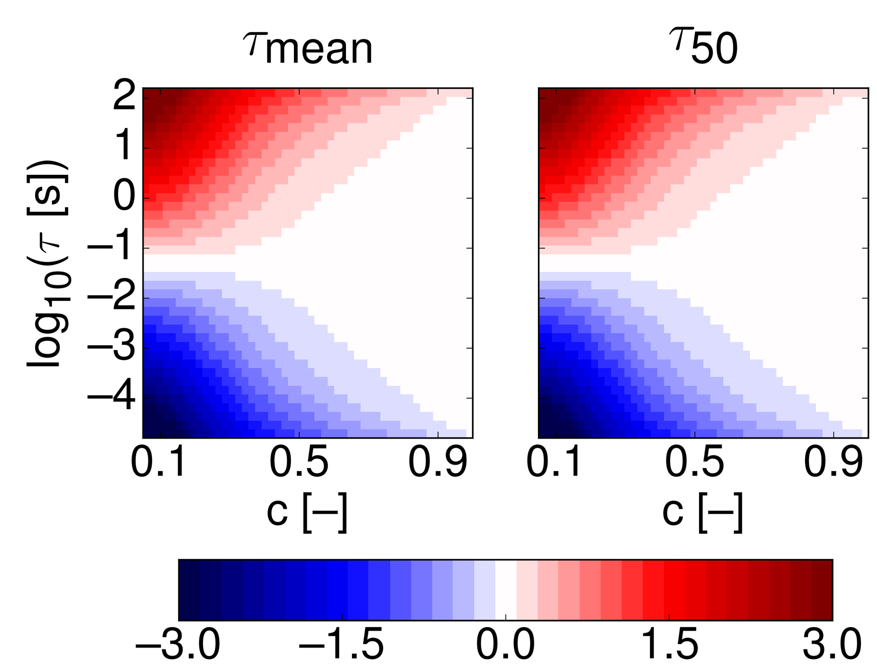
Reconstruction of m_{tot}



The total chargeability m_{tot} increasingly underestimates the original CC based m parameter with decreasing c value and with τ approaching the relaxation time limits. This behavior is observed for different CC kernels and is compressed within the range $c \leq \tilde{c}$.

Reconstruction of relaxation times





The deviation between input and recovered relaxation time shows a similar pattern as observed for the chargeability. Deviations of up to three orders of magnitude exist for decreasing c values and τ approaching the relaxation time limits.

Reconstruction patterns are similar for τ_{mean} and τ_{50}.

Summary

- Cole-Cole chargeability is underestimated by up to 80%, when determined using a CCD
- Characteristic relaxation times of the CC model and CCD differ by up to three orders of magnitude
- CC parameters can only be estimated by CCD parameters for mono-modal SIP signatures when τ_{mean} and τ_{50} are used

These results highlight the importance of a consistent SIP data analysis procedure, especially if results from different studies are to be quantitatively compared.

Software

The CCD software is maintained under:

`https://github.com/m-weigand/Debye_Decomposition_Tools`

- Resistivity and conductivity formulations
- Open-source
- Python 2.7

References

Pelton, W., Ward, S., Hallof, P., Sill, W., and Nelson, P. (1978). Mineral discrimination and removal of inductive coupling with multifrequency ip. *Geophysics*, 43(3):588–609.

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