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Fast Excitation Magnetic field of MRS Computation for layered earth

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SUMMARY

We have developed a fast numerical algorithm for computing the excitation magnetic field distribution of MRS for layered earth. We first give the expressions of the MRS response in layered earth, and then obtain the unknown parameters in the expressions using the boundary conditions. We use a new method named quadrature-with-extrapolation (QWE) to solve the problem of dual Bessel function's calculation when the integral is divergent. We validate the results of applying our algorithm against the commercial software, and apply our method to a number of MRS synthetic examples. Surprisingly, our method is faster than the commercial software for all layered models, and the accuracy is acceptable.

Key words: MRS, layered model, dual Bessel function, quadrature-with-extrapolation (QWE).

INTRODUCTION

Magnetic Resonance Sounding (MRS) is used for groundwater exploration to map water content and hydraulic conductivity. The resistivity of the subsurface has great effect on the electromagnetic field, then to affect the signal of MRS. Homogeneous half-space model is used widely in the traditional MRS kernel computation, and the kernel is precomputed and stored for later use in MRS inversion. However, with the rapid development of joint inversion of MRS and other electrical and electromagnetic methods, a fast algorithm of MRS modelling for layered earth is needed urgently.

In this paper, we first collect the expressions of the MRS response in layered earth, and get the unknown parameters in the expressions using the boundary conditions. With the iterative method, the whole expressions can be obtained.

The expression relies on evaluating integrals of the form

$$F(r)=\int_0^\infty f(k)g(kr)dr$$

Where $g(kr)$ is an oscillatory dual Bessel function. The term $f(k)$ is the kernel function that may also be oscillatory. The integral can be resolved by two methods, including fast Hankel transform (FHT) and the standard quadrature method. The FHT can obtain highly speed, but it failed to handle the divergent integrals. The standard quadrature method can be slow to converge or may also fail if the integral is divergent. Consequently, special care is required for their numerical evaluation.

To solve the problem of the divergent integral in MRS modelling, a new method called quadrature-with-extrapolation (QWE) is developed. A simple Matlab implementation of the fast MRS modelling algorithm is provided and is compared with the commercial software. As shortly revealed, the results suggest that our method is faster than the commercial software for all layered models, and the accuracy is acceptable.

METHOD AND RESULTS

The amplitude of MRS signal is

$$E_0(q)=\int_0^l [K(q,\rho,z)\cdot \mathrm{w}(z)]dz \quad (1)$$

$$K(q,\rho(z),z)=\frac{2\pi f_L M_0}{I_0}\iint_{x,y} B_{1\perp}(\rho(z),z)\sin(\frac{\gamma q B_{1\perp}(\rho(z),z)}{2})dxdy \quad (2)$$

where $\mathrm{w}(z)$ is the distribution of water content, $\rho(z)$ is the distribution of resistivity; M_0 is the magnetization; $\gamma=2.675\times10^8\mathrm{s}^{-1}\mathrm{T}^{-1}$ is the hydrogen proton gyromagnetic ratio; $B_{1\perp}$ is the geomagnetic vertical component of the excitation magnetic field in layered earth; $K(q,\rho(z),z)$ is the kernel function, which represents the sensitivity of the receiver coil to the underground water and E_0 is the initial amplitude of the MRS signal.

The general solution of Hertz potential is:

$$F_j=\frac{I_0 a}{2}\int_0^\infty \frac{J_1(\lambda a)}{\lambda}[a_j e^{-u_j z}+b_j e^{u_j z}]J_0(\lambda r)d\lambda \quad (3)$$

Where $j=0,1,2,...,n$; $u_j=\sqrt{\lambda^2+k_j^2}$; a_j and b_j are unknown parameters. To obtain the unknown parameters $a_1,a_2,...,a_j,...a_n$ and $b_0,b_1,...,b_j,...b_{n-1}$ in the expression, we need to use the boundary conditions of F at the interface to established an equation set made up of $2n$ equations. Make the dielectric permeability of each layer the same, we can have the relations:

$$\begin{cases} F_j=F_{j+1} \\ \dfrac{\partial F_j}{\partial z}=\dfrac{\partial F_{j+1}}{\partial z} \end{cases} \quad (4)$$

In the air $a_0 = 1$ and in the base $b_j = 0$. Bring the formula (3) into dielectric boundary conditions (4), we can obtain the following equations satisfied by the unknown parameters.

The surface:

$$1 + b_0 = a_1 + b_1 \tag{5}$$

$$-\lambda + \lambda b_0 = -u_1 a_1 + u_1 b_1 \tag{6}$$

The j-th interface ($j = 1, 2, ..., n-2$):

$$a_j e^{-u_j h_j} + b_j e^{u_j h_j} = a_{j+1} e^{-u_{j+1} h_j} + b_{j+1} e^{u_{j+1} h_j} \tag{7}$$

$$u_j a_j e^{-u_j h_j} - u_j b_j e^{u_j h_j} = u_{j+1} a_{j+1} e^{-u_{j+1} h_j} - u_{j+1} b_{j+1} e^{u_{j+1} h_j} \tag{8}$$

The top face of the base:

$$a_{n-1} e^{-u_{n-1} h_{n-1}} + b_{n-1} e^{u_{n-1} h_{n-1}} = a_n e^{-u_n h_{n-1}} \tag{9}$$

$$u_{n-1} a_{n-1} e^{-u_{n-1} h_{n-1}} - u_{n-1} b_{n-1} e^{u_{n-1} h_{n-1}} = u_n a_n e^{-u_n h_{n-1}} \tag{10}$$

We found that the relationship of the unknown parameters a_j and b_j of each layer, as well as the a_j and a_{j+1} of adjacent layer are linear, so it is much simple to solve the problem, and the specific formulas are as follows:

$$z_j = -i\omega\mu_0 / u_j \tag{11}$$

$$z^{(j)} = z_j \frac{z^{(j+1)} + z_j th(u_j H_j)}{z_j + z^{(j+1)} th(u_j H_j)} \tag{12}$$

$$z^{(n)} = z_n \tag{13}$$

$$f_j = \frac{z^{(j+1)} - z_j}{z^{(j+1)} + z_j} \tag{14}$$

$$G_{j,j-1} = \frac{1 + f_j}{1 + f_j e^{-2u_j h_j}} e^{(u_j - u_{j-1}) h_{j-1}} \tag{15}$$

$$a_j = \begin{cases} 1 & , j = 0 \\ G_{j,j-1} a_{j-1} & , j = 1, 2, ..., n\text{-}1 \\ (1 + f_{n-1}) e^{(u_n - u_{n-1}) h_{n-1}} a_{n-1} & , j = n \end{cases} \tag{16}$$

$$b_j = \begin{cases} \dfrac{z^{(1)} - z_0}{z^{(1)} + z_0} & , j = 0 \\ f_j e^{-2u_j h_j} a_j & , j = 1, 2, ..., n\text{-}1 \\ 0 & , j = n \end{cases} \tag{17}$$

Now, the whole expression of (3) can be given. We can obtain the radial component H_r and vertical component H_z of the magnetic field at $P(r, z)$ in the j-th medium:

$$H_{rj}(r, z) = \frac{I_0 a}{2} \int_0^\infty [a_j e^{-u_j z} - b_j e^{u_j z}] u_j J_1(\lambda r) J_1(\lambda a) d\lambda \tag{18}$$

$$H_{zj}(r, z) = \frac{I_0 a}{2} \int_0^\infty [a_j e^{-u_j z} + b_j e^{u_j z}] \lambda J_0(\lambda r) J_1(\lambda a) d\lambda \tag{19}$$

The formula above contains the product of two Bessel functions and has a divergent term. To solve this problem, the formula can be shown as the following form:

$$H(\lambda) = \int_0^\infty f(k) g(k\lambda) d\lambda = \sum_{i=0}^{\infty} F_i \tag{20}$$

Where

$$F_i = \int_{k_{i-1}}^{k_i} f(k) g(k\lambda) d\lambda \tag{21}$$

I use the zeros of the oscillatory function g as the interval breakpoints $k_{i-1} < k_i$. F_i can be evaluated using Gauss quadrature rule

$$F_i \approx \sum_{j=1}^{m} w_j f(x_j / \lambda) g(x_j) \tag{22}$$

Where m is the Gauss quadrature order and w are weights associated with the quadrature abscissae x.

Similar to the FHT method, the formula (18) and (19) can be rearranged to

$$\begin{aligned} F_i &\approx \sum_{j=1}^{m} w_j f(x_j / \lambda) g(x_j) = \sum_{j=1}^{m} f(x_j / \lambda) \hat{g}(x_j) \\ &= f(X / \lambda)^T \hat{g}(X_n) = S_n \end{aligned} \tag{23}$$

Where S_n is the direct partial sum. $\hat{g}(x_j) = w_j g(x_j)$ is independent of the λ, so it can be precomputed and stored for later use. According to the QWE method mentioned in the reference 8, S_n can be obtained. So, after the iteration, the problem of expression (18) and (19) can be solved.

Figures and Tables

Take a five-layer model for example, the first layer is resistive, with the resistivity of $500\,\Omega\,\text{m}$, and the thickness of $20\,\text{m}$; the second layer is the main aquifer, with the resistivity of $100\,\Omega\,\text{m}$, the thickness of $20\,\text{m}$, and the water content of 20%; The third layer is the aquifuge, with the resistivity of $500\,\Omega\,\text{m}$, and the thickness of $20\,\text{m}$; The fourth layer is a minor aquifer, with the resistivity of $100\,\Omega\,\text{m}$, the thickness of $20\,\text{m}$, and the water content of 10%; And the resistivity of the basement is $1000\,\Omega\,\text{m}$. Fig 1 is the result of Hr and Hz calculated by the algorithm above (red coil) and the commercial software (blue line). We can see that there is great consistency between the two. And surprisingly, the QWE method took much less time than the commercial software.

Fig 2 is the MRS kernel and initial amplitude of the above model, given by the QWE method.

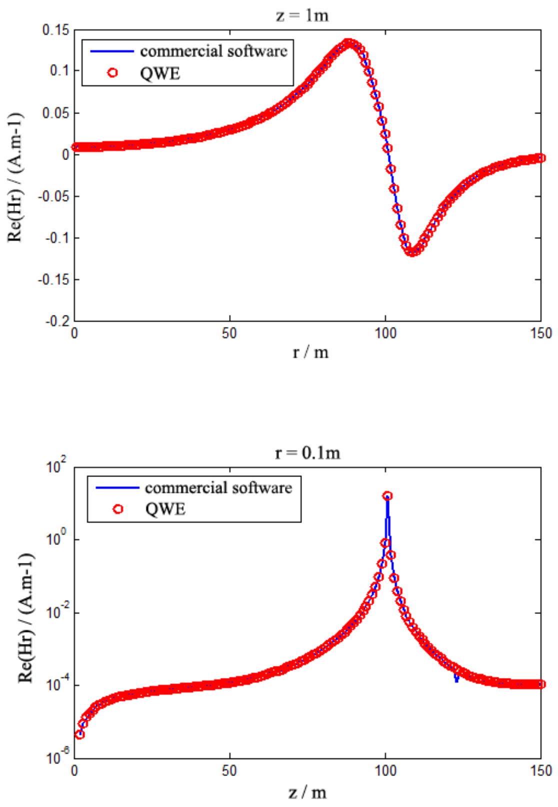

Figure 1. Hr and Hz calculated by the QWE method (red coil) and the commercial software (blue line).

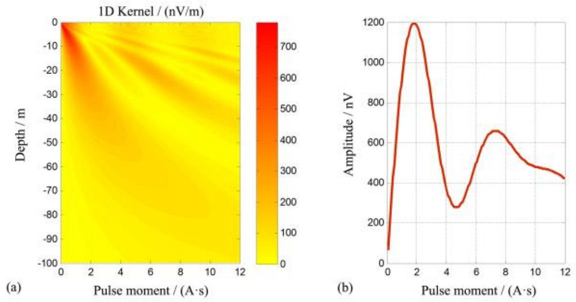

Figure 2. MRS forward result using the QWE method (a) MRS kernel (b) initial amplitude.

CONCLUSIONS

In this paper we have given a simple way to calculate the excitation magnetic field of MRS in layered earth, and also introduced a fast method of MRS forward. We validated the results of applying our algorithm against the commercial software, and applied our method to a number of MRS synthetic examples. We can see that (1) the method to get the parameters in the expressions of the magnetic field is effective, (2) the QWE method can solve the divergent integration problem with great speed and accuracy, and can be employed in MRS forward. The fast algorithm in this paper will be a base of MRS inversion.

REFERENCES

Mohnke, O., and Yaramanci, U., 2002, Smooth and block inversion of surface NMR amplitudes and decay times using simulated annealing: Journal of Applied Geophysics, 50, 163-177.

Weichman, P.B., Lavely, E.M., and Ritzwoller, M., 1999, Surface nuclear magnetic resonance imaging of large systems: Physical Review Letters ,82, 4102-4105.

Weichman, P.B., Lavely, E.M., and Ritzwoller, M., 2000, Theory of surface nuclear magnetic resonance with applications to geophysical imaging problems: Physical Review, 62, 1290-1312.

Hertrich, M., Braun, M., and Yaramanci, U., 2005, Magnetic resonance soundings with separated transmitter and receiver loops: Near Surface Geophysics, 3, 131-144.

Hertrich, M., Braun, M., Günther, T., Green, A.G., and Yaramanci, U., 2007, Surface Nuclear Magnetic Resonance Tomography: IEEE Transactions on Geoscience and Remote Sensing, 45, 3752-3759.

Hertrich, M., and Yaramanci, U., 2002, Joint inversion of Surface Nuclear Magnetic Resonance and Vertical Electrical Sounding: Journal of Applied Geophysics, 50, 179-191.

Kong, F.N., 2007, Hankel transform filters for dipole antenna radiation in a conductive medium: Geophysical prospecting, 55, 83-89.

Kerry, K., 2012, Is the fast Hankel transform faster than quadrature?: Geophysics, 77, 21-30.

Jochen, A., Lehmann, H., Marian, H., Greenhalgh, S.A., and Alan, G.G., 2011, Three-Dimensional Magnetic Field and NMR Sensitivity Computations Incorporating Conductivity Anomalies and Variable-Surface Topography: IEEE TRANSACTIONS ON GEOSCIENCE AND REMOTE SENSING, 49, 3878-3891.