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An Assessment of the Relationship for Estimating Hydraulic Conductivity from NMR Measurements in Unconsolidated Sediments

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SUMMARY

Nuclear magnetic resonance (NMR) logging provides a new means of estimating the permeability (k) or hydraulic conductivity (K), of unconsolidated aquifers. The estimation of K from the measured NMR parameters can be accomplished using the Schlumberger-Doll Research (SDR) equation. The SDR equation includes empirically-determined constants. Decades of research for petroleum applications have resulted in standard values for these constants that can provide accurate estimates of permeability in consolidated formations for petroleum applications. This study examines whether similar global constants can be derived for near-surface applications that would yield accurate estimates of K in unconsolidated aquifers. We apply four new methods of analysis to re-analyze data from three field sites in North America to determine the range of values for the SDR constants that fit the data. We show that all the wells in our dataset can be explained by a single value or narrow range of values for the constants. We also show that the porosity term can be removed from the SDR model without significantly affecting the quality of our fit to the data. These results suggest that we can define standard constants that can be used to obtain high resolution, cost-effective estimates of K from NMR logging in unconsolidated aquifers. The methodologies we employ provide an effective approach to evaluating the performance of the SDR model.

Key words: NMR, SDR, hydraulic conductivity, logging, uncertainty

INTRODUCTION

The evaluation, management, and protection of our groundwater resources requires an ability to obtain reliable estimates of subsurface properties that control the movement of fluids in the near-surface region (up to ~100 m below the surface). One such property is hydraulic conductivity (K) or permeability (k), which describe the ease with which water flows in the subsurface. Surface or logging nuclear magnetic resonance (NMR) provide a means of estimating these parameters in the saturated zone of the near-surface (Dlubac et al., 2013; Walsh et al., 2013). The development of reliable tools for measuring NMR has necessitated the simultaneous development of a model that will relate permeability or hydraulic conductivity to the NMR measurement. The petroleum industry has already adopted the Schlumberger-Doll Research (SDR) equation (Kenyon et al., 1988) as a way to estimate permeability from logging NMR measurements of the relaxation time constant T_2. The SDR equation contains empirical constants, and significant research has led to standard values for these constants in consolidated sandstones, materials of interest to industry. Reasonable accuracy can be obtained by using these constants without a need for local calibration. However, recent studies have shown that different constants are needed for unconsolidated sediments (Dlubac et al., 2013). Knight et al. (2015) examined this question further and specifically looked at multiple near-surface sites where both permeability and logging NMR measurements were available and was able to show that three different sites with very different lithologies could be modeled with similar constants. The purpose of the current study is to re-visit the data used by Knight et al. (2015) to better understand the relationship between T_2 and K. The methodologies employed provide a useful approach to evaluating the performance of the model as well as exploring the total space of allowable parameter values for the model. While we analyze logging NMR data, we anticipate that the results will be applicable to the interpretation of surface NMR T_2 data as well.

METHOD AND RESULTS

We re-analyze the data from the extensive field surveys conducted by Knight et al. (2015). NMR and hydraulic conductivity data were obtained from three sites, GEMS2 (Kansas), Larned (Kansas), and Leque Island (Washington state). These sites and their characteristics and lithologies are fully described in Knight et al. (2015). There are a total of 112 independent measurements of both permeability and NMR measurements, distributed in 9 wells. The entire dataset is plotted in Figure 1.

We use a number of different methods to re-analyze the data from Knight et al. (accepted) in order to understand various aspects of the data and model, as well as to understand the importance of the various parameters involved in the SDR equation and the range of uncertainty in these parameters. We want to estimate the uncertainty in the constants in our model, instead of optimizing for the best-fitting values, because these are likely to be biased by noisy data and imperfect models.

The model we consider is the Schlumberger-Doll Research (SDR) equation for predicting permeability from T_{2ML} and porosity (ϕ), given by

$$K_{SDR} = b\phi^m \left(T_{2ML}\right)^n \quad (1)$$

where b, n, and m are empirical constants that can be optimized to provide the best fit between predicted and measured permeability (e.g. Kenyon et al., 1988). The model is based on the Kozeny-Carmen relationship (Carmen, 1956; Kozeny, 1927). In this form the equation is non-linear and thus would present a challenge to find the best-fitting parameters. However, we can transform the equation into logarithmic space, which turns it into a linear problem

$$\log(K_{SDR}) = \log_{10}(b) + m\log_{10}(\phi) + n\log_{10}(\mathrm{T}_{2ML}) \quad (2)$$

This is the equation for a plane in 3-dimensional log space, with $\log_{10}(T_{2ML})$ and $\log_{10}(\phi)$ acting as the independent variables and $\log_{10}(k)$ the dependent variable. This formulation has the benefit that extreme values of permeability are equally weighted and it is completely linear. We will work strictly with the logarithmic form of the SDR equation for this study.

The four methods we use are a stepwise analytical linear regression (ALR), parameter grid search, bootstrap, and Markov Chain Monte Carlo (MCMC) using Bayesian statistics. The linear regression is an analytical machine learning algorithm. The grid search works by plotting a residual associated with a large number of value-pairs for two parameters and identifying which value pairs have a low residual. Bootstrap and MCMC are both Monte Carlo (sampling) approaches that use a large number of iterations to construct density functions that are used to estimate the range of parameters that fit the data. Bootstrap works by repeatedly selecting a subset of the data with replacement and optimizing for the best-fitting parameters (Parsekian et al., 2014). MCMC evaluates the likelihood of thousands of parameter values and builds a distribution based on which values are more likely given the data.

We present the results for each of the empirical constants in the SDR equation below (*m*, *n*, and *b*), as well as the data errors estimated from the MCMC inversion. Our results are only valid for estimating hydraulic conductivity using the linearized, log-space Equation (2).

NMR-derived estimates of porosity (ϕ) do not contribute significantly to predicting permeability in unconsolidated sediments. We find that porosity acts essentially like a noisy constant. We find that porosity does not improve the prediction of *K* in our data. We can thus simplify the SDR equation (Equations 1 & 2) to include just the mean log decay time, T_{2ML}. We can express this modified SDR equation as the "Knight-Maurer" equation for predicting hydraulic conductivity in unconsolidated sediments:

$$K_{KM} = b\left(T_{2ML}\right)^{n} \quad (3)$$

Of course, any model with more free parameters will always fit the data at least as well or better than a model with fewer parameters, but for predictive value, fewer parameters often perform better. Because we want to estimate permeability at many sites other than those we use to derive the parameter values, having a simpler model that does equally well is a strong advantage.

Figure 1 shows results for *n* on each well. Most of the wells give values close to 1 or 2. This is consistent with the expectation that *n* indicates the diffusion regime of the location, with *n* = 1 corresponding to a slow diffusion regime and *n* = 2 corresponding to a fast diffusion regime. We also found using MCMC that if we fix the value of n to either 1 or 2 we can still fit the data within errors at all sites. The maximum likelihood estimates are around 1.7 for the entire dataset, indicating that there is likely a mixture of both fast- and slow-diffusing sites within our dataset.

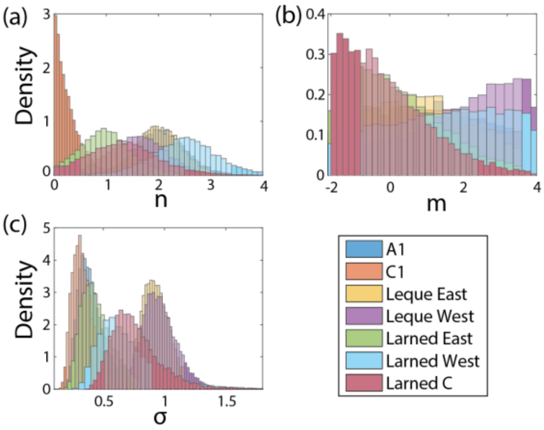

Figure 1. MCMC Results for m, n, and σ estimated for each well in our datset. *n* is the exponent on T_{2ML} and *m* is the exponent on porosity in the SDR equation (Equation 2) for each of the wells in our dataset. (a) Distributions on *n*. Notice that one site, C1, has a maximum at *n* = 0. This site is characterized by a small range in hydraulic conductivity and high noise, and fits perfectly within a scheme with n = 1 or 2 when combined with the other data, thus is not a robust result. (b) Distributions on *m*. Note that in some cases, slightly negative values are preferred, some slightly positive, and overall there is no significant trend similar to that for *n*. This is evidence that porosity is not really improving the prediction; i.e. we are merely fitting noise when we include porosity. Setting *m* = 0 gives us almost as good a fit at every site and makes for a simpler model. (c) Distributions on $\sigma(K)$. Estimating the error in the data *given our model* is very important for understanding how well we match the data. An error of ~1 (log of hydraulic conductivity) indicates that we only resolve hydraulic conductivity to within an order of magnitude. The results shown here are for each individual site; for the whole site we get an average error of approximately 0.75 log units.

Once we have fixed the values of *m* (=0) and *n* (we use *n* = 2 for the remainder of this study), we can solve for the value of and uncertainty in *b*. Figure 2 shows the results for four different methods for estimating b: (a) MCMC, (b) bootstrap, (c) ALR, and (d) directly estimating from the data for each site. The ranges using MCMC and bootstrap overlap significantly, indicating that the data do not require multiple *b*-values for each site. The range in *b* is greater for the ALR and direct methods, but this is due to noisy data (direct) and incorrect Gaussian assumptions (ALR). Even so, the ranges for these methods overlap significantly and have mean values close to that of the other methods, around $10^{-1.5} \approx 0.0316$, with the range in most cases extending from 10^{-1} – $10^{-2.5}$ (0.1 – 0.00316). It appears that a value for *b* chosen from this range should predict *K* reasonably well for any near-surface (unconsolidated) field site. This is a surprising result, but consistent with Knight et al., 2015.

Finally, we use the MCMC method to estimate the data standard error, *σ*, given our model and an assumed covariance structure. While off-diagonal terms in the covariance matrix probably exist in reality, we assume a diagonal covariance matrix and compute the scalar weight *σ* that fits the data using the MCMC algorithm. This is found to be approximately 0.75

(log hydraulic conductivity units) for the full data set and approximately 0.5 for the individual wells.

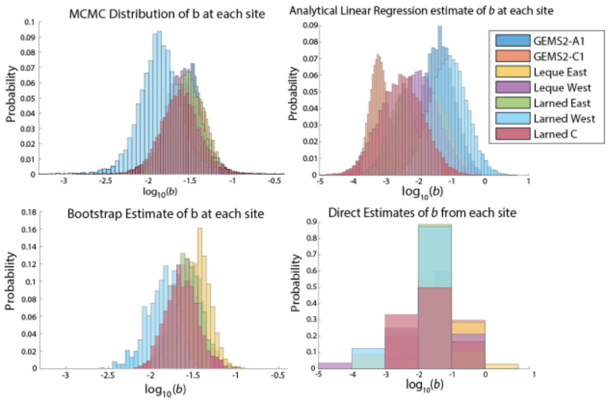

Figure 2. Results from four different methods for estimating b in the modified SDR equation (Equation 2) for each of the wells in our dataset. For these inversions we assume *m* = 0 and *n* = 2.

CONCLUSIONS

In this study we have analyzed data collected at three different field sites, each with different lithologies, and analyzed it using four different methods. Our overall results are consistent across all methods and field sites, indicating that our results be applicable to more than just our data set. We have shown that it should be possible to use NMR data and a modified SDR equation with a fixed set of constants to predict hydraulic conductivity in unconsolidated sediments, at least to within an order of magnitude. We have shown that in unconsolidated sediments, porosity does not improve the prediction of permeability within our data set, and we assume that this extends to all unconsolidated sediments. We propose a set of parameter values that can be used to estimate *K* at all sites with unconsolidated sediments. Our results for *b* are consistent with the findings of Knight et al., 2015. Table 1 summarizes how our results compare to those of Knight et al. (2015) as well as standard values used by industry.

We have taken four different approaches to the problem of determining parameter values for the SDR equation. These methods are easily applicable to other studies, and represent a fundamentally different approach than simply optimizing data to find the best-fitting parameter values. We propose that researchers collecting new surface or logging NMR data as well as independent estimates of permeability would benefit from applying these methods to their study. We have also simplified the SDR equation by removing the dependence on porosity. This should give greater predictability at different sites, since there are fewer empirical constants to tune. Based on our results, it should be possible for researchers working at sites with unconsolidated sediments to predict *K* to within an order of magnitude using NMR estimates of T_2.

Table 1. *b*-values for the SDR equation from this study and others. Units are 10^{-2} m/s^3.

	m = 4	m = 1	m = 0
Our Study	80 – 470	5 – 11	1.5-3.6
Knight et al., 2015	80 - 570	5 - 12	
Typical Industry	3.4		
Industry Range	3.4 - 4.25		

REFERENCES

Carmen, P.C., 1956, Flow of Gases Through Porous Media: Academic Press Inc., New York.

Dlubac, K., R. Knight, Y. Song, N. Bachman, B. Grau, J. Cannia, J. Williams. 2013. The use of NMR logging to obtain estimates of hydraulic conductivity in the high plains aquifer, Nebraska, USA. *Water Resources Research*. DOI: 10.1002/wrcr.20151.

Kenyon, W.E., Day, P.I., Straley, C., and Willemsen, J.F., 1988, A Three-Part Study of NMR Longitudinal Relaxation Properties of Water-Saturated Sandstones: SPE-15643-PA, doi: 10.2118/15643-PA.

Knight, R., D.O. Walsh, .J. Butler, Jr., E. Grunewald, G. Liu, A.D. Parsekian, E.C. Reboulet, S. Knobbe, and M. Barrows, 2015 (accepted manuscript), A New Approach for Estimating Hydraulic Conductivity in Unconsolidated Aquifers.

Kozeny, J., 1927, Uber kapillare Leitung des Wassers im Boden: Sitzungsber. Akad. Wiss. Wien, v. 136, p. 271–306.

Parsekian, A.D., Dlubac, K., Grunewald, E., Butler, J.J., Knight, R., and Walsh, D.O., 2014, Bootstrap Calibration and Uncertainty Estimation of Downhole NMR Hydraulic Conductivity Estimates in an Unconsolidated Aquifer: Ground Water, doi: 10.1111/gwat.12165.

Walsh, D., Turner, P., Grunewald, E., Zhang, H., Butler, J.J., Reboulet, E., Knobble, S., Christy, T., Lane, J.W., Johnson, C.D., Munday, T., and Fitzpatrick, A., 2013, A small-diameter NMR logging tool for groundwater investigations: Ground Water, v. 51, no. 6, p. 914–926.