MRS2015 / 6th International Workshop on Magnetic Resonance

Inversion of magnetic resonance data considering varying geomagnetic field

Anatoly Legchenko
IRD, LTHE,
BP53, 38041, Grenoble, France
Anatoli.Legtchenko@ird.fr

Jean-Michel Vouillamoz
IRD, LTHE,
08BP841 Cotonou, Bénin
jean-michel.vouillamoz@ird.fr

Fabrice Messan Amene Lawson
University of Abomey Calavi,
Cotonou, Bénin
lawson.amen@yahoo.fr

Christian ALLE
University of Abomey Calavi
Cotonou, Bénin
christian.alle@ird.fr

Marc Descloitres
IRD, LTHE,
BP53, 38041, Grenoble, France
marc.descloitres@ird.fr

Marie Boucher
IRD, LTHE,
08BP841 Cotonou, Bénin
Marie.Boucher@ird.fr

SUMMARY

It is known that at the scale of MRS field setup the Earth's magnetic field may vary in space and in time. These variations are caused by different natural factors and cannot be compensated by accurate tuning of the measuring devise. Varying geomagnetic field causes non-resonance conditions of excitation that affect both amplitude and phase of the MRS response. Usually variations of the Earth's magnetic field do not exceed a few hertz and their effect on the amplitude is relatively small permitting to assume a constant geomagnetic field for inversion. However, under some specific conditions both the amplitude and the phase may vary sufficiently for rendering inversion erroneous if the off-resonance conditions are not taken into account. We have developed and tested a new algorithm of inversion of MRS measurements considering varying Earth's magnetic field. We tested this approach using synthetic and field data and we have found that inversion was improved. For demonstration purposes we present inversion of MRS data measured in Benin (Western Africa) with time-varying Earth's magnetic field. Because of improved modeling of the phase shift the newly developed algorithm allows to use inversion of complex signals, which allows better resolution than inversion of the amplitudes. We show that inversion of complex signals provided inverse model better corresponding to the ground truth.

Key words: SNMR, MRS, inversion, Benin, GRIBA.

INTRODUCTION

Magnetic Resonance Sounding (MRS) measurements are performed in the Earth's magnetic field B_0, which is usually considered as constant for given area. However, the geomagnetic field is not always constant. Depending on magnetic properties of surrounding rocks the geomagnetic field may be perturbed locally at the pore size scale or it may gradually change its intensity with depth (Roy *et al.*, 2008). The Earth's magnetic field may also vary during measuring time (Vouillamoz *et al.*, 2008). Varying geomagnetic field modify the Larmor frequency thus creating off-resonance conditions of excitation, which affects magnetic resonance signal (Legchenko *et al.*, 1997; Legchenko, 2004; Hertrich, 2008) and consequently these variations should be taken into account for the inversion. However, these effects are usually considered as relatively small and in the majority of cases they are neglected.

Information about variations of the Earth's magnetic field in the subsurface at the MRS loop scale is not easy available. For example, measurements with a magnetometer on the surface provide only general idea about Earth's magnetic field in rocks and measurements in borehole are not always available and they are also local. Fortunately, MRS signal contains information about the geomagnetic field (Larmor frequency) that can be extracted from measured signals trough the inversion procedure. We developed and tested on synthetic and real data an algorithm of non-linear 1-D inversion of MRS data that takes into account time and depth variations of the Earth's magnetic field. Inversion is carried out iteratively recomputing the kernel considering off-resonance conditions derived from MRS signal.

Modeling results show that the algorithm converges relatively fast and provides reliable results. The forward modeling shows that when variations of the Earth's magnetic field are small and water saturated formation is thin the effect of the off-resonance excitation on the inversion results is relatively small. Otherwise, knowledge of the Earth's magnetic field variations was found necessary. A large equivalence between time and depth variations of the geomagnetic field and consequently of the Larmor frequency renders necessary determination of the cause of the Larmor frequency variations. We demonstrate the advantage of using this algorithm on field data measured in Western Africa (Benin).

METHOD

For computing MRS signal we assume one coincident Tx/Rx loop and the frequency offset $\Delta\omega$. Performing FID1 measurements with one current pulse we can compute the received signal decaying with the relaxation time T_2^* as

$$e(q,t) = I_0^{-1}\int_V \omega_0 B_1 e^{2\varphi_0^{Tx}} M_\perp e^{\varphi_{\Delta\omega}} w(\mathbf{r}) \times \qquad (1)$$
$$\times e^{-t/T_2^*(\mathbf{r})} \times e^{j\omega_0 t} dV,$$

where $q=I_0\tau_p$ is the pulse moment with I_0 and τ_p being the amplitude and duration of the current pulse in the loop;

$B_1(\rho(\mathbf{r}))$ is the component of the loop magnetic field perpendicular to the Earth's magnetic field; $\phi_0^{Tx}(\rho(\mathbf{r}))$ is the phase shift caused by the subsurface and $\mathbf{r}$ is the coordinate vector. $M_\perp$ is the perpendicular component of the nuclear magnetization computed taking into account $\Delta\omega(\mathbf{r})$ being the frequency offset between the Larmor frequency $\omega_0(\mathbf{r})$ and the pulse frequency ω and is corresponding phase shift $\phi_{\Delta\omega}$ (Legchenko, 2004). Measured signal oscillates with the Larmor frequency and decays with the time constant $T_2^*(\mathbf{r})$.

In Equation (1), $e(q,t)$ is a ,set of experimental data and unknown functions of interest are: $w(\mathbf{r})$ – the water content, $\rho(\mathbf{r})$ – electrical resistivity of the subsurface, $B_0(\mathbf{r}) = \omega_0(\mathbf{r})/\gamma$ – the Earth's magnetic field (γ is the gyromagnetic ratio) and the relaxation time $T_2^*(\mathbf{r})$.

Resolution of the Equation (1) could be done straightforward using a global non-linear inversion. However we found that such an approach requires a high signal to noise ratio (S/N) and is computationally difficult. For simplifying inversion we do the following:

1) It has been shown by Braun and Yaramanci (2008) that the resistivity $\rho(\mathbf{r})$ can be resolved from inversion of MRS data set. However, the resistivity can be also measured by one of the well-developed geophysical methods that provide better resolution in comparison with MRS inversion. For example, it has been shown that uncertainty in time-domain electromagnetic method (TDEM) results does not influence MRS inversion for water content (Legchenko *et al*., 2008). Thus we assume in Equation (1) $\rho(\mathbf{r})$ to be known from other measurements.

2) We split inversion of the MRS signal $e(q,t)$ into two parts: inversion versus time $e(q=const,t)$ and inversion versus pulse moment $e(q,t=const)$.

3) Different inversion schemes for the relaxation time $T_2^*(\mathbf{r})$ inversion can be found in the literature (Legchenko and Valla, 2002; Mohnke and Yaramanci, 2002; 2005; Mueller-Petke and Yaramanci, 2010). We use the time-step-inversion (TSI), which allows separating inversion for $T_2^*(\mathbf{r})$ from inversion of $e(q,t=const)$ for water content.

4) When performing inversion for $\omega_0(\mathbf{r})$ we assume one average value of the Larmor frequency for each value of the pulse moment thus transforming recorded time series into $\omega_0(q)$ data set using the Fourier transform. As only the maximum of the spectra amplitude corresponding to the Larmor frequency at given pulse moment is computed, the shape of the spectra closely related to $T_2^*(\mathbf{r})$ has no significant influence on $\omega_0(q)$ determination.

5) In this paper we assume 1-D inversion. $T_2^*(z)$ is defined by the TSI inversion with fixed $\omega_0(z)$, which is derived from the iterative inversion for $w(z)$ and $\omega_0(z)$.

Our algorithm is split into two parts: a linear inversion for the water content and a non-linear inversion for the Larmor frequency. Both parts are linked by an iterative procedure presented in Figure (1).

For linear inversion Equation (1) is approximated by the matrix equation $\mathbf{Aw} = \mathbf{e}$ and then resolved using the Tikhonov regularization method (Legtchenko and Shushakov, 1998). For computing the matrix $\mathbf{A}$ we use known distribution of the Larmor frequency iteratively derived from the non-linear inversion for $\omega_0(z)$.

The matrix $\mathbf{A}$ can be computed considering either time or depth variations in the geomagnetic field. In the first case slow variations of the Larmor frequency during measuring time can be monitored with a proton magnetometer or derived from measured SNMR signal (time of the day for each pulse moment is recorded by the instrument) and then converted into variation versus pulse moment: $\omega_0(t)\rightarrow\omega_0(q)$. Then, the MRS response for each pulse moment $q=q_i$ is computed using the same value of the Larmor frequency $\omega_0(q_i)$ for all depths. In the second case a non-linear inversion for $\omega_0(\mathbf{r})$ is required for converting $\omega_0(q)\rightarrow\omega_0(z)$. Then, MRS response is computed for all pulse moments considering $\omega_0(z_j)$ for corresponding depth $z=z_j$. The non-linear inversion is performed using non-linear least square optimization (Marquardt, 1963).

Inversion stops when the residual between experimental and measured data computed with individual weights for frequency, amplitude and phase (P_ω, P_e, and P_ϕ) becomes smaller than the noise level.

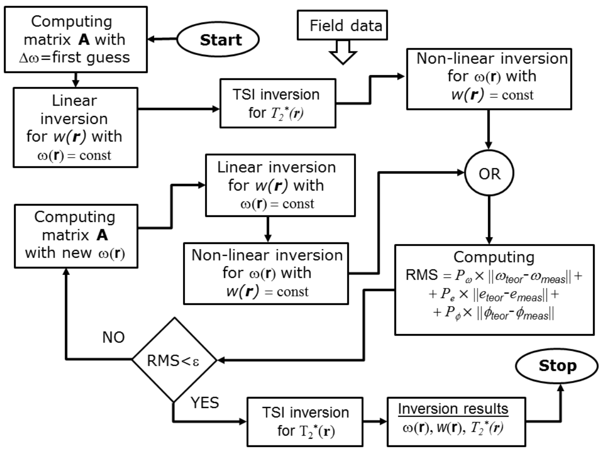

Figure 1. Flowchart of the inversion algorithm adapted to varying geomagnetic field.

RESULTS

As mentioned above, it is important to identify the origin of the Larmor frequency variation. We demonstrate it using set of synthetic data. Signals are computed for two cases: 1) the frequency offset is varying linearly from the depth of 0 to 100 m (0-10 Hz); 2) the frequency offset is varying linearly from the first to the last pulse moment, which corresponds to time varying geomagnetic field (also from 0 to 10 Hz). These signals were compared with the case computed assuming constant the geomagnetic field. For computing we assume a 100×100 m^2 square loop, 100 ohm-m half-space, water-saturated layer from 0 to 100 m with w=20% and measuring conditions typical for Europe.

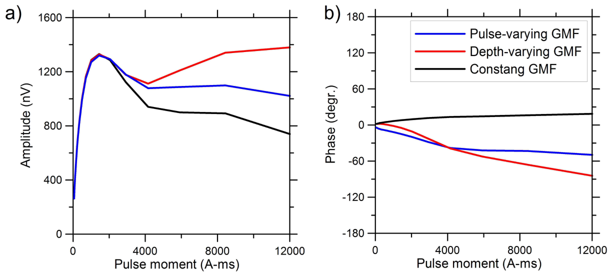

Figure 2. Amplitude (a) and phase (b) of MRS response computed considering one water-saturated layer between 0 and 100 m (*w*=20%). Three cases are presented: constant geomagnetic field (black line), depth-varying geomagnetic field with the frequency offset between 0 and 10 Hz (red line) and pulse-moment varying geomagnetic field also between 0 and 10 Hz (blue line).

One can see (Figure 2) that for smaller pulse moments the difference is small because of the small frequency offset. For larger pulse moments however the frequency offset goes to 10 Hz and the difference in the MRS responses increases. Consequently, if an incorrect model was chosen then the corresponding errors should be expected in the inverse model.

Practical implementation of the above described inversion algorithm to inversion of synthetic data did no revealed significant mathematical difficulties and convergence was reasonably fast. The inversion time was largely dependent on time necessary for computing the matrix $\mathbf{A}$.

Presented approach was used for investigating groundwater resources in Benin. This study has been carried out in the framework of GRIBA project (EuropeAid program). The subsurface in the investigated site is composed of hard rock (mainly gneiss), which is weathered down to about 30 m. Borehole drilled in this area showed that this weathered part represents an aquifer down to 32 m with the static water level at 5.3 m. An Electrical Resistivity Tomography (ERT) profile (Figure 3a) shows that this aquifer formation was well resolved by ERT due to a low electrical resistivity (about 50 ohm-m) of water-saturated weathered rock relative underlying hard rock (more than 1000 ohm-m).

MRS station is located in the area corresponding to the distance between 200 and 300 m along the profile. For MRS survey we used NUMIS^{plus} system with a 50×50 m^2 square loop. The Larmor frequency was about 1415 Hz. The origin of observed variations of the Larmor frequency was verified by measuring MRS signal with the same pulse moment in the beginning and at the end of the sounding. It was established that we deal with the time variations of the Earth's magnetic field. This result was confirmed by attempts of measuring spin echo (SE) signals (Legchenko *et al*., 2010). The absence of SE corroborate with the geology composed of non-magnetic gneiss. These observations allow selecting for inversion a model with the time-varying geomagnetic field.

Inversion of complex signals shows that the aquifer was well resolved (Figure 3b) and MRS inverse model is in a good agreement with ERT. Measured amplitude and phase (*E(q)* and $\phi(q)$) are well fitted by the theoretical signal computed after inversion results. However, when the amplitude inversion was carried out assuming a constant geomagnetic field we obtain less accurate inverse model (Figure 3c). The inverse model shows reasonable results for the shallow part of the subsurface. Below 30 m however, inversion suggests a water-saturated formation non-confirmed by ERT and borehole. In this case inversion of complex signals was inaccurate because of poor fit of the phase due to an inadequate mathematical model.

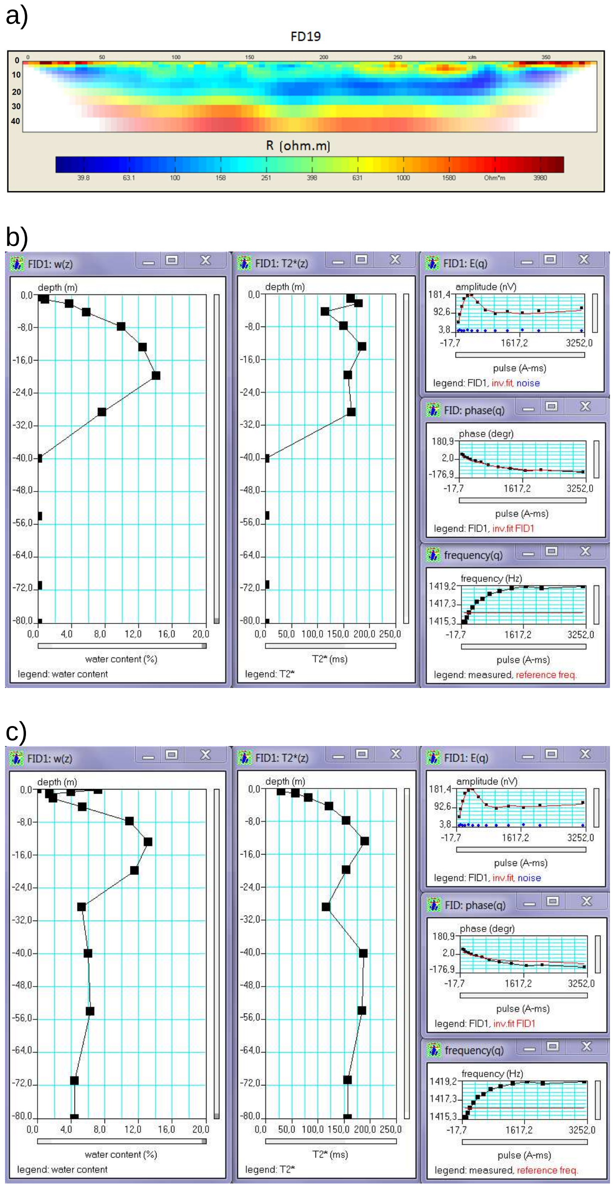

Figure 3. Field study of an aquifer in Benin: a) ERT profile; b) MRS inversion carried out considering variations of the geomagnetic field; c) MRS inversion carried out considering constant geomagnetic field.

CONCLUSIONS

We developed and tested an algorithm for inversion of MRS data measured in a non-constant geomagnetic field. Numerical modeling and field verifications show that the inversion is reliable but requires preliminary identification of the origin of observed variations of the Larmor frequency.

The possibility to use inversion of complex signals allows improving accuracy of the recovered inverse model and potentially may improve inversion of MRS data for resistivity distribution as suggested by Braun and Yaramanci (2008).

ACKNOWLEDGMENTS

Presented results were obtained in the framework of the GRIBA project (Groundwater Resources In Basement rocks of Africa) funded by The African Union, The European Union, and the Institut de Recherche pour le Développement (grant AURG/098/2012). The content of this paper is the sole responsibility of the authors and can under no circumstances be regarded as reflecting the position of The European Union or The African Union. This work also benefited from the support of the AQUI BENIN JEAI project supported by IRD.

REFERENCES

Braun, M., and Yaramanci, U., 2008, Inversion of resistivity in Magnetic Resonance Sounding: Journal of Applied Geophysics 66, 151–164, doi:10.1016/j.jappgeo.2007.12.004.

Hertrich, M., 2008, Imaging of groundwater with nuclear magnetic resonance: Progress in Nuclear Magnetic Resonance Spectroscopy, 53, 227–248, doi: 10.1016/j.pnmrs.2008.01.002.

Legchenko, A.V., and Shushakov, O.A., 1998, Inversion of surface NMR data: Geophysics, Vol. 63, 75-84, doi: 10.1190/1.1444329.

Legchenko, A.V., Beauce, A., Guillen, A., Valla, P., and Bernard, J., 1997, Natural variations in the magnetic resonance signal used in PMR groundwater prospecting from the surface: European Journal of Environmental and Engineering Geophysics, 2, 173-190.

Legchenko, A., and Valla, P., 2002, A review of the basic principles for proton magnetic resonance sounding measurements: Journal of Applied Geophysics, 50, 3–19, http://dx.doi.org/10.1016/S0926-9851(02)00127-1.

Legchenko, A., 2004, Magnetic Resonance Sounding: Enhanced Modeling of a Phase Shift: Applied Magnetic Resonance, 25, 621-636, doi: 10.1007/BF03166553.

Legchenko, A., Ezersky, M., Girard, J.-F., Baltassat, J.-M., Boucher, M., Camerlynck, C., Al-Zoubi, A., 2008, Interpretation of magnetic resonance soundings in rocks with high electrical conductivity: Journal of Applied Geophysics, 66, 118-127, doi:10.1016/j.jappgeo.2008.04.002.

Legchenko, A., Vouillamoz, J.M., and Roy J., 2010, Application of the magnetic resonance sounding method to the investigation of aquifers in the presence of magnetic materials: Geophysics, 75 (6), L91–L100, doi: 10.1190/1.3494596.

Marquardt, D., 1963, An algorithm for least-squares estimation of non-linear parameters: Journal of the Society for Industrial and Applied Mathematics, 11, 431-441, doi: http://dx.doi.org/10.1090/S0025-5718-2011-02496-8.

Mohnke, O., and Yaramanci, U., 2002, Smooth and block inversion of surface NMR amplitudes and decay times using simulated annealing: Journal of Applied Geophysics 50, 163–177, doi: 10.1016/S0926-9851(02)00137-4.

Mohnke, O., and Yaramanci. U., 2005, Forward modeling and inversion of MRS relaxation signals using multiexponential decomposition: Near Surface Geophysics, 3, 165–85, doi: 10.3997/1873-0604.2005012.

Mueller-Petke, M., and Yaramanci, U., 2010, QT-inversion-comprehensive use of the complete surface-NMR data set: Geophysics, 75, WA199–WA209, doi: 10.1190/1.3471523.

Roy, J., Rouleau, A., Chouteau, M., and Bureau, M., 2008, Widespread occurrence of aquifers currently undetectable with the MRS technique in the Grenville geological province, Canada: Journal of Applied Geophysics, 66, 82–93, doi: 10.1016/j.jappgeo.2008.04.006.

Vouillamoz, J.M., Favreau, G., Massuel, S., Boucher, M., Nazoumou, Y., Legchenko, A., 2008, Contribution of magnetic resonance sounding to aquifer characterization and recharge estimate in semiarid Niger: Journal of Applied Geophysics, 64, 99-108, doi:10.1016/j.jappgeo.2007.12.006.