A New Method for Parameter Estimation of Magnetic Resonance Sounding

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SUMMARY

Magnetic resonance sounding (MRS) allows for a direct, non-invasive and in-situ determination of the water content of the surface, accurate parameter estimation is the key to a successful MRS survey. The initial phase φ of free induction decay (FID) as well as phase and amplitude of power line harmonics are obtained by Fast Fourier Transform (FFT), power line harmonics are subtracted from MRS signal. Cross-correlation is proposed to suppress the random noise and spikes, in order to reconstruct FID signal, an approximation formula is given. Estimation of MRS initial amplitude E_0 and spin-spin relaxation time T_2^* by using a Duffing oscillator is investigated. Simulation results show that the proposed method has a relative error 1.8%, 6% and 1.7% in φ, E_0 and T_2^* respectively when SNR is as low as –32 dB.

Key words: MRS; Parameter estimation; cross-correlation; Duffing oscillator

INTRODUCTION

Parameter estimation is made for the amplitude, phase, relaxation time T_2^* of the FID signal from where a water content and a permeability image is derived, inaccurate parameter estimation makes MRS results unreliable, it is hence of great value to estimate the parameters accurately. Today, the main obstacle of parameter estimation is the low SNR of MRS records. The conventional idea of parameter estimation is envelop detection: linear or non-linear regression curve-fitting technique based on least squares estimation procedures. A SNR higher than 0 dB makes an accurate fitting easy to perform. Generally, the SNR of MRS measurements in fields is less than 0 dB. Therefore, band-pass filtering, mean or weighted stacking, de-spiking and adaptive noise cancelling (ANC) (Dalgaard et al., 2012), etc, have been proposed to improve the SNR. Selection of the filtering technical depends on the noise origin, so a variety of filters are used in conjunction when the MRS signal is corrupted by multiple noises, these procedures can be more or less efficient, but some part of non-filtering noise is always remaining in the records. What's more, the above methods have corresponding drawbacks: band-pass filter reduces the noise and may distort the FID signal at the same time; stacking is effective but time-consuming; ANC is efficient only when the primary signal and the reference signal is highly linearly correlated and multichannel instrumentation is needed.

In this paper, a new method for parameter estimation is proposed, which extracts parameters indirectly rather than fits a filtered and smooth FID, which means there is no need to make the FID waveform visible in time-series. The FFT can obtain the initial phase of FID as well as amplitude and phase of power line harmonics with a lower SNR, following that power line harmonics are constructed and subtracted from the MRS signal. Cross-correlation is an effective method to suppress random noise and spikes, an approximation formula defining the relationship between FID and cross-correlation signal is given. Duffing oscillator is sensitive to certain periodic signal but immune to noise, this property can be applied to detect signal with low SNR. Parameter estimation of FID based on the state transition of the driven Duffing oscillator is presented. Finally, the results of simulations are present.

METHOD

The normal form of the Duffing equation is defined as:

$$\ddot{x}(t) + \eta\dot{x}(t) - x(t) + x^3(t) = a\cos(\omega t)$$

where η denotes coefficient of viscous damping, η=0.5 in this paper, a is the amplitude of the driving force, the dot denotes differentiation with respect to time. To detect weak signals with arbitrary frequency ω, through frequency transformation we can obtain:

$$\ddot{x}(t) = -\omega\eta\dot{x}(t) + \omega^2[x(t) - x^3(t) + a\cos(\omega t)]$$

According to Duffing equation, the simulation model is shown in Fig. 1, G0=ω^2, G1= $\eta\omega$, G2=1/ω. The driving force or the signal is input into the Simulink through *Simin* from workspace.

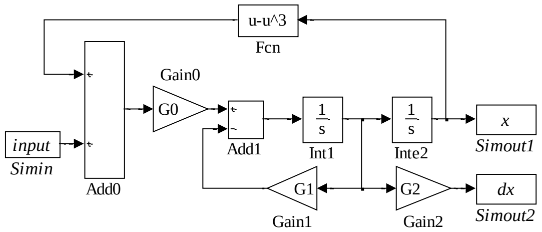

Figure 1. Simulation model of Duffing oscillator

As the driving force decreases from 1 to 0, the system will experience five states: (1) periodic motion, (2) chaotic motion, (3) period doubling bifurcation (4) homoclinic orbit, and (5) attractor, which are shown in Fig. 2.

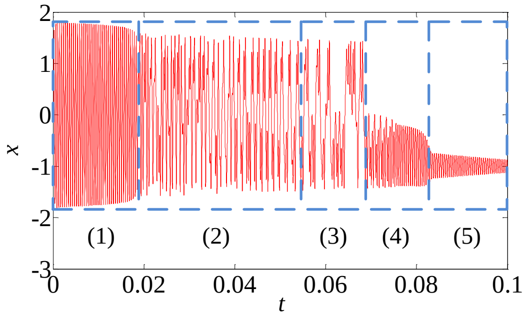

Figure 2. Output of Duffing oscillator system when the amplitude of driving force decreases, *a*=0.8(1-t/0.12)

There are four critical states among five states, but only two state transitions are easy to determine: from periodic motion to chaotic motion and from homoclinic orbit to attractor. Threshold values of system states transition are determined by the system itself, the two threshold values obtained by simulation are list in Tab. 1

Table 1. Thresholds of phase transition of Duffing system

State transition	value
a_{12}(periodic motion to chaotic)	0.7156
a_{45}(homoclinic orbit to attractor)	0.2467

When the system is in a critical state, even a tiny perturbation may cause a qualitative change of the system state, only certain periodic signal does but not do the noise (Wang et al, 1999), which can be illustrated in Fig. 3.

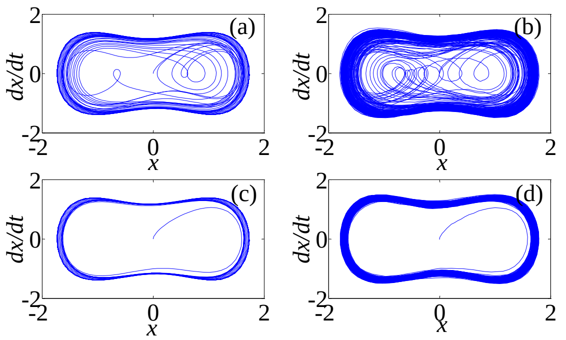

Figure 3. Phase plane diagrams. (a) *a*= 0.7156, chaotic, (b) *a*= 0.7156, with noise, chaotic, (c) *a*=0.7157, periodic, (d) *a*= 0.7157, with noise, periodic.

In Fig.3, the standard deviation of random noise is 0.001. When the system was in the critical state, the system was still in chaotic motion after the noise was input; while the periodic signal with amplitude of 0.0001 and frequency of ω was input, the system transformed into periodic motion, and the noise cannot change the state back to the chaotic motion. It shows that Duffing oscillator is sensitive to certain periodic signal but immune to noise, thus the Duffing oscillator potential to detect the periodic signal with damped amplitude and SNR of -20 dB, which is much lower than the thresholds of the traditional time domain detection method.

Estimation algorithm

The FID signal is given by the following expressions:

$$s(k) = E_0 \exp(-\frac{k}{f_s T_2^*})\cos(\frac{\omega k}{f_s} + \varphi) + n(k),\ k = 1,2\cdots N$$

where $f_0=\omega/2\pi$ is Larmor frequency, f_s is sampling frequency, N is number of samples. The entire parameter estimation workflow for MRS signals is as follows:

(1) FFT and Estimate φ;
(2) Power line harmonics canceling
(3) Cross-correlation
(4) Reconstruct FID
(5) Estimate E_0 and T_2^* by chaotic detection

(1) As we know, FFT returns information in the form of a complex vector whose length is the same with numbers of samples, thus we can obtain magnitude and phase of sinusoids (power line harmonics) from the corresponding complex element. Although FID is not a standard sinusoid, its phase can still be extracted accurately, especially when sampling frequency is an integral multiple of Larmor frequency. The initial phase of FID is given as:

$$\hat{\varphi} = \arctan(\frac{\mathrm{im}(X(Nf_0/f_s))}{\mathrm{re}(X(Nf_0/f_s))})$$

(2) When the frequencies of power lines harmonics are in close proximity to f_0 (Δf < 50Hz), the similarity between them will deteriorate the cross-correlation, so it is better to reduce them before cross-correlation. Notch filter is a good solution, but it may dampen the FID. We can obtain the amplitude and phase of the power line harmonics via FFT, and then we reconstruct them and subtract them from recorded signal. For example, Larmor frequency is 2320 Hz, noise at frequency of f_1 (2300Hz) and f_2 (2350Hz) should be subtracted, which can be written as:

$$\begin{cases} y(k) = u(k) - a_1\cos(\frac{2\pi f_1 k}{f_s} + \theta_1) - a_2\cos(\frac{2\pi f_1 k}{f_s} + \theta_2) \\ a_1 = \mathrm{abs}(X(Nf_1/f_s)),\ a_2 = \mathrm{abs}(X(Nf2/f_s)) \\ \theta_1 = \arctan((\frac{\mathrm{im}(X(Nf_1/f_s))}{\mathrm{re}(X(Nf_1/f_s))})),\theta_2 = \arctan((\frac{\mathrm{im}(X(Nf_2/f_s))}{\mathrm{re}(X(Nf_2/f_s))})) \end{cases}$$

$u(k)$ denotes the time-series recorded in field. If Nf_1/f_2 is not an integer, spectral leakage will happen. In that case, two closest frequency signals should be subtracted.

(3) Cross-correlation is a measure of similarity of two series as a function of the lag of one relative to the other, cross-correlation with a sinusoid can also be used to recuperate signal waveform corrupted by uncorrelated noise. FID signal with noise and reference sinusoid are calculated through cross-correlation function to improve SNR.

$$\frac{1}{N}\sum_{n=0}^{N-1} 2\cos(\frac{2\pi f_0(n-k)}{f_s})y(n) = R(k) + R_n(k)$$

$2\cos(2\pi f_0 t)$ is the reference signal. Take advantage of the irrelevancy of correlation between signal & noise as well as noise & noise, FID can be enhanced and noise can be restrained through cross-correlation algorithm.

(4) Because the FID is not a standard sinusoids signal, cross-correlation function $R(k)$ is quite complicated, in order to compute it quickly, the approximation formula is given as:

$$\begin{cases} s(k) = \exp(-\frac{k}{2f_s T_2^*})R(k) + \frac{2a_0}{10}\exp(-\frac{k}{f_s T_2^*})\cos(\frac{\omega k}{fs} + \varphi) + \varepsilon(k) \\ a_0 = \mathrm{abs}(X(Nf_0/f_s)) \end{cases}$$

where $\varepsilon(k)$ denotes error, $R(k)$ is a damped sinusoid with frequency ω. When f_s=64 f_0, the variance of error is illustrated in Fig. 4.

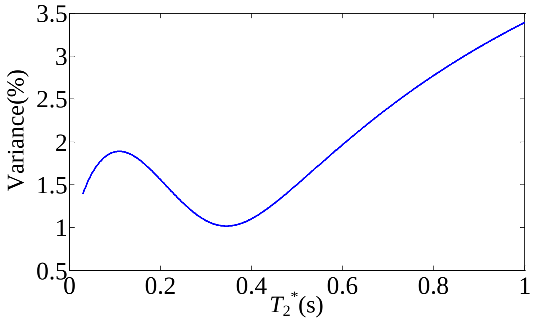

Figure 4. Variance of approximation formula with different relaxation time T_2^*

From Fig. 4, we can find the maximum variance is 3.3% when T_2^*equals 1000 ms, variance is less than 2% when T_2^* on an interval [0.03 06], the accuracy is sufficient to estimate parameter of FID. After cross-correlation, the FID signal is reconstructed by the approximation formula.

By subtracting power line harmonics and cross-correlation, the SNR will be improved significantly. $s(k)$ is input into the Duffing oscillator as driving force, by determining the time when the output has a state transition from periodic motion to chaotic motion and another state transition from homoclinic orbit to attractor, we can estimate the parameters of FID:

$$\begin{cases} a_{12} = \lambda E_0 \exp(-\frac{t_{12}}{T_2^*}), \\ a_{45} = \lambda E_0 \exp(-\frac{t_{45}}{T_2^*}) \end{cases}$$

Because the initial amplitude of MRS signal is at nV level, λ is used to make FID strong enough to set the Duffing oscillator in the periodic state at first and in the attractor state at last. The estimated parameters are written as:

$$\begin{cases} \hat{T}_2^* = (t_{45} - t_{12}) / (\ln a_{12} - \ln a_{45}) \\ \hat{E}_0 = a_{12} \exp(T_2^* / t_{12}) / \lambda \end{cases}$$

SIMULATION RESULTS

Simulations had been performed where a MRS signal was added by noise, the signal was written as:

$$s(t) = 1 \times 10^{-7} \exp(\text{-t/0.3})*\cos(2\pi * 2326 * t + \pi / 6)$$

The FID initial amplitude was 100 nV, phase was π/6, and relaxation time was 300 ms. Noise consists of random noise, spikes and power line harmonics, the noise was written as:

$$n(t) = \text{wgn}(1, N, -120) + 10^{-5}\,\text{pulstran(t, 0.05:0.05:0.5, 'gauspuls')}$$

$$+ 2 \times 10^{-7} \cos\left(2\pi * 2300 * t + \frac{\pi}{4}\right) + 2 \times 10^{-7} \cos\left(2\pi * 2350 * t + \frac{\pi}{3}\right)$$

where wgn(1,N,-120) is a random noise with zero mean and variance 10^{-12}, pulstran function is used to generate spikes. Simulation time was 500 ms and sampling frequency was 148864 Hz. The time-series and spectrum of $s(t)$+$n(t)$ is shown in Fig. 5 & Fig. 6.

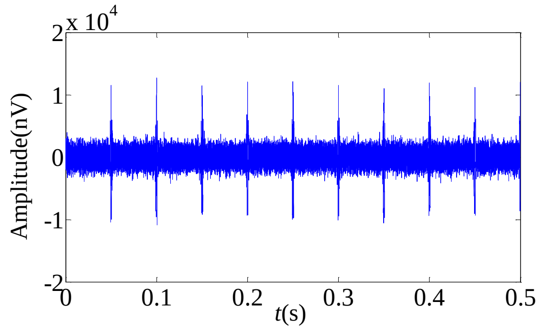

Figure 5. Time-series of MRS

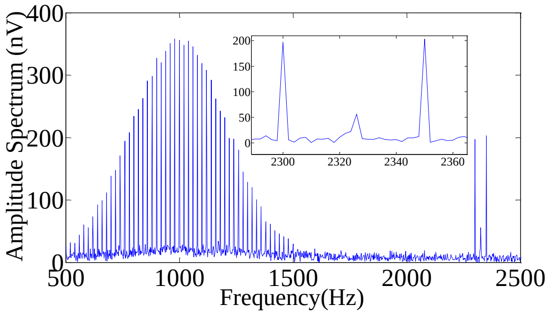

Figure 6. Spectrum of MRS

As we can see, the FID was completely corrupted in the noise in time-series, but still can be found in spectrum even though it was very weak. The SNR was calculated as:

$$\text{SNR} = 10 * \log\left(\frac{\sum s(t)^2}{\sum (s(t) + n(t))^2}\right)$$

the calculated SNR of simulation signal was -32 dB.

The estimated initial phase of FID, amplitude and phase of power line harmonics through FFT are list in Tab.2.

Table 2. Estimates of initial phase of FID

Frequency(Hz)	Amplitude(nV)	Phase(degree)	Phase Error(%)
2300	211.5	46.29	+2.8%
2326	45.27(a_0)	**29.45**	-1.8%
2350	205	60.70	+1.2%

The estimated phase was 29.45° with relative error of 1.8%. Then the power line harmonics were subtracted, spectrum of filtered data is shown is Fig.7. As we can see, the power line harmonics are removed completely.

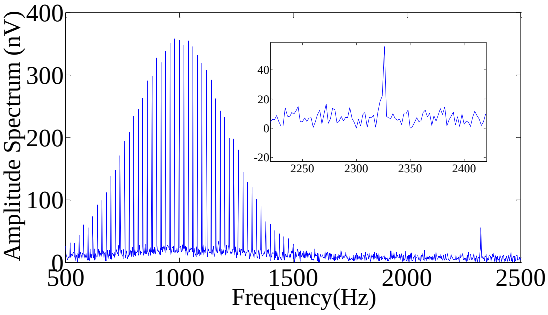

Figure 7. Spectrum of MRS after subtracting power line harmonics

The filtered data was conduct cross-correlation with 2cos(ωt), which could be done by the *xcorr* function in Matlab. Through cross-correlation, most of the noise will be remove to the beginning part of the cross-correlation series. The reconstructed FID is shown in Fig. 8, its fractional waveform is shown in Fig. 9. The difference between them is quite small.

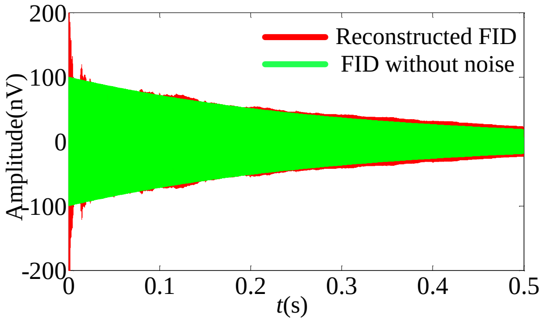

Figure 8. Reconstructed MRS signal and FID without noise

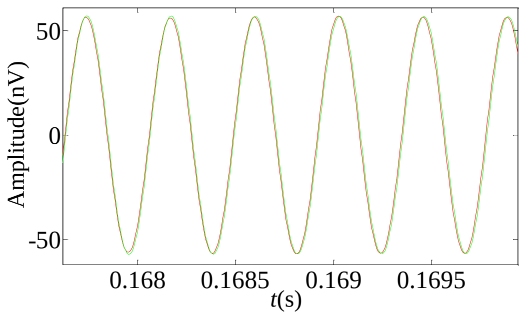

Figure 9. Fractional waveform comparison between reconstructed MRS (red) and FID without noise (green)

Fig. 10 is the phase plane of the output, which denotes that the system experience periodic motion and chaotic motion.

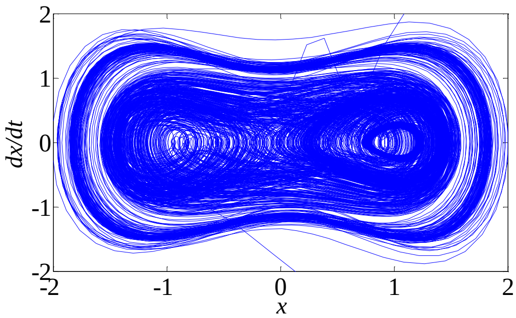

Figure 10. Phase plane diagrams.

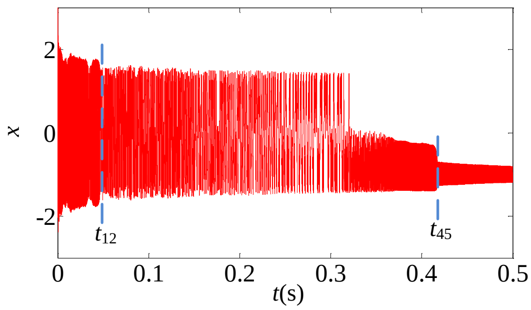

Figure 11. Output of Duffing oscillator system when MRS signal is input

Fig. 11 shows the output of the system, from which we can determine the t_{12}= 94.34 ms, t_{45}= 408.1ms. Finally, the estimated parameters of MRS is E_0=93.9 nV, T_2^*=295 ms, the relative error is 6% and 1.7% respectively.

CONCLUSIONS

We have presented a new method to estimate the parameters of MRS signal with Duffing oscillator. First, we introduce the fundamental principle of Duffing oscillator detection, which is capable of detecting damped periodic signal with low SNR. Though FFT the initial phase of FID is estimated meanwhile is parameters of power line harmonics are obtained. With the amplitude and phase of power line harmonics, we can subtract them from MRS signal. Cross-correlation is used to suppress the spikes and random noise, after which a smooth FID is reconstructed, the error of approximation formula is less than 3.3%. Finally, T_2^* and E_0 are estimated by determining the time when the state of Duffing oscillator transforms. The parameter estimation has a relative error 1.8%, 6% and 1.7% in φ, E_0 and T_2^* respectively when the SNR is as low as -32 dB. Thus the SNR threshold of parameter estimation of the MRS signal is reduced significantly. Although this method is powerless when the noise is at the same frequency with FID (maybe reference noise cancelling is the only choice), it is a potential solution to allow the application of MRS in noisy areas and improve the investigation depth & resolution.

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