A preliminary research of 2D surface-NMR tomography based on coincident loops

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SUMMARY

Surface Nuclear Magnetic Resonance (SNMR) is a relatively new geophysical method for groundwater exploration and aquifer characterization. So far, the practical application of SNMR technique is mostly based on 1-D forward and inversion strategies, which are inappropriate for investigating isolated water occurrences and using in complicated hydrological environments. What's more, most of the inversions only employ the real part of the voltage response data. In order to expand the application scope and inversion resolution of SNMR method, we investigate a 2-D forward and inversion scheme considering the elliptical polarization and phase lag effect. After calculating the transmitted oscillating magnetic field by Chave algorithm, we construct 4 synthetic aquifer models and obtain their forward complex response. Then we compare and analysis the inversion results of applying (1) real data, (2) imaginary data and (3) joint with real and imaginary data through Marquardt method for these models. By numerical simulation, we find that the Marquardt can reconstruct the original models correctly and the imaginary part data is illustrated to be more sensitive to 2D dimension structures than the real part of the signal. Joint inversion not only stabilizes the inversion process but also improves resolution, so we should make the best of imaginary part data if it is possible, especially for exploration in complicated hydrological environments.

Key words: SNMR, joint inversion, tomography, elliptical polarization.

INTRODUCTION

Groundwater is an essential water resource for human society, which plays an important role in agricultural irrigation, industrial and mining enterprises, and urban life, especially in the surface water shortage areas, groundwater becomes the main source of water supply. So we need more efficient ways to search for and manage groundwater in a sustainable fashion (Hertrich et al., 2007).

Currently, among all the geophysical techniques which are available for groundwater exploration, only SNMR can directly supply quantitative information about the locations and amounts of free water in the subsurface. The early forward and inversion theory of SNMR are based on 1-D layered hypothesis theory with coincident coil, without considering the influence of aquifer on subsurface conductivity and regarding the resistivity of underground medium as homogenous half space or layered model. In the past 20 years, many scientists have conducted the 1-D forward and inversion research (Legchenko and Shushakov, 1998; Mohnke and Yaramanci, 2002; Yaramanci and Mueller-Petke,2009). At present, 1-D theory of SNMR is relatively matured, the practical exploration is also mainly based on 1-D explanation. However the actual groundwater storage structures are usually two or even three-dimensional, so regarding all the underground water-bearing structure as 1-D layered model will produce large errors, and even get wrong results. Following mathematical formulations of Weichman et al. (2000), some effort has been made to extend to two dimensions.

This paper is also a preliminary research of 2-D SNMR tomography and assessing the potential for recovering high resolution 2-D images of water content using both real and imaginary voltage response data. After summarizing the formulas for computing the NMR response of earth models containing free water, we describe our inversion strategy briefly. Finally, we compare and analysis the inversion results of applying (1) real data, (2) imaginary data and (3) joint with real and imaginary data for different aquifer models (perched water lens, 2-D dipping layer model, paleochannel and karst caverns).

BASIC THEORY

The SNMR technique exploits the intrinsic nuclear spins of protons in groundwater, which precess at the characteristic Larmor frequency of the earth's magnetic field $\mathbf{B}_0$. The first step of SNMR measurement involves stimulating the spin magnetic moments to tilt with respect to the earth's magnetic field by transmitting a current through a large loop deployed on the earth's surface. The current alternates at the Larmor precession frequency and usually has a pulse duration τ_p of the order of 40ms. After the transmitter current is completely terminated, the spins continue precessing about the earth's magnetic field, generating a secondary magnetic field which can be detected by a surface receiver loop.

As we know, only the perpendicular component of the excitation field to the local earth's magnetic field $\mathbf{B}_0$ interacts with the spin system. When the conductivity of subsurface medium is high, the perpendicular excitation field are elliptically polarized and it may be decomposed into two circularly polarized real fields, the co- and counter-rotating part (Figure 1). What's more, the NMR signal will be complex and the two parts are unequal to each other at the most time and has different effects respectively (Weichman et al., 2000).

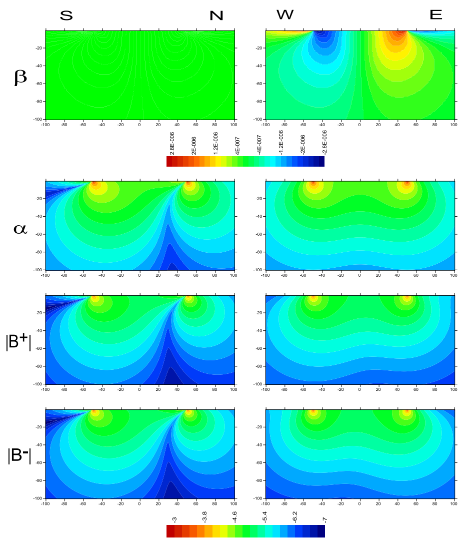

Figure 1: South-North (left column) and West-East (right column) sections of the elliptically polarized parameters β_T (semi-minor axis),α_T (semi-major axis), B^+_T and B^-_T for a circular loop of 100m, 50Ω·m half-space.The earth magnetic field is 48000nT at an inclination of 60◦.

For the general case of separate transmitter and receiver loops at arbitrary positions, the initial maximum voltage response of a surface loop after extinction of the pulse is given by

$$V_0(q)=\omega_L \mathbf{M}_0 \int d^3 r f(\mathbf{r}) \sin(-\gamma \frac{q}{I_0}|\mathbf{B}^+_\mathbf{T}(\mathbf{r})|)$$
$$\times \frac{2}{I_0}|\mathbf{B}^-_\mathbf{R}(\mathbf{r})| \bullet e^{i[\zeta_T(\mathbf{r},\omega_L)+\zeta_R(\mathbf{r},\omega_L)]} \tag{1}$$
$$\times[\hat{\mathbf{b}}^\perp_\mathbf{R}(\mathbf{r},\omega_L)\bullet \hat{\mathbf{b}}^\perp_\mathbf{T}(\mathbf{r},\omega_L)+i\,\hat{\mathbf{b}}_\mathbf{0}\bullet \hat{\mathbf{b}}^\perp_\mathbf{R}(\mathbf{r},\omega_L)\times \hat{\mathbf{b}}^\perp_\mathbf{T}(\mathbf{r},\omega_L)]$$

where $\mathbf{M}_0$ is the specific magnetization of hydrogen protons, ω_L is the angular Larmor frequency , $f(\mathbf{r})$ is free-water content in the volume at location r, γ is the gyromagnetic ratio of hydrogen protons, $\mathbf{B}^+_\mathbf{T}(\mathbf{r})$ is the co-rotating component of the transmitter field rotates clockwise around $\mathbf{B}_0$, and $\mathbf{B}^-_\mathbf{R}(\mathbf{r})$ is the counter-rotating component of the virtual receiver field that would be observed if the receiver loop were used as a transmitter loop. The terms ζ_T and ζ_R are phase lags associated with the distances between the two loops and points r in the subsurface. The unit vectors $\hat{\mathbf{b}}^\perp_\mathbf{T}$, $\hat{\mathbf{b}}^\perp_\mathbf{R}$ and $\hat{\mathbf{b}}_0$ are the orientations of the transmitter and virtual receiver fields and the earth's magnetic field at each volume element, respectively (Marian, 2005).

Neglecting the attenuation of the received voltage response at short times after the transmitted signal, the voltage response V can be written in the form (Weichman et al., 2000)

$$V(t)=\mathrm{Re}(V_0 e^{-i(\omega_L t+\varphi)})$$
$$=V_R \cos(\omega_L t+\varphi)+V_I \sin(\omega_L t+\varphi) \tag{2}$$

Where φ is the initial phase of the transmitted signal, V_R and V_I are the real and imaginary data which we are used to do joint inversion.

For convenient representation of the SNMR response, the kernel of the integral usually defined as

$$K(\mathrm{q},\mathbf{r})=\omega_L \mathbf{M}_0 \sin(-\gamma \frac{q}{I_0}|\mathbf{B}^+_\mathbf{T}(\mathbf{r})|)$$
$$\times \frac{2}{I_0}|\mathbf{B}^-_\mathbf{R}(\mathbf{r})| \bullet e^{i[\zeta_T(\mathbf{r},\omega_L)+\zeta_R(\mathbf{r},\omega_L)]} \tag{3}$$
$$\times[\hat{\mathbf{b}}^\perp_\mathbf{R}(\mathbf{r},\omega_L)\bullet \hat{\mathbf{b}}^\perp_\mathbf{T}(\mathbf{r},\omega_L)+i\,\hat{\mathbf{b}}_\mathbf{0}\bullet \hat{\mathbf{b}}^\perp_\mathbf{R}(\mathbf{r},\omega_L)\times \hat{\mathbf{b}}^\perp_\mathbf{T}(\mathbf{r},\omega_L)]$$

So the voltage response can be represented as

$$E_0(q)=\int K(q,\mathbf{r})\, f(\mathbf{r})\, d^3 r \tag{4}$$

When the transmitter and receiver loops coincide (3) reduces to the form

$$K(\mathrm{q},\mathbf{r})=\frac{2}{I_0}\omega_L M_0 \sin(-\gamma \frac{q}{I_0}|\mathbf{B}^+_\mathbf{T}(\mathbf{r})|)\times|\mathbf{B}^-_\mathbf{R}(\mathbf{r})| \bullet e^{2i\zeta(\mathbf{r})} \tag{5}$$

The form of equation (4) is identical to that for coincident loops (Hertrich et al., 2009).

For 2D conditions, if the system extends to infinity in the $\pm y$ directions i.e., $\partial f(y)/\partial y=0$.The 2-D kernel function can be obtained by

$$K_{2D}(q;x,z)=\int_{-\infty}^{\infty} K_{3D}(q;x,y,z) \tag{6}$$

And the voltage response can be written as

$$E_0(q)=\int_0^\infty \int_{-\infty}^\infty K_{2D}(q;x,z)\cdot f(x,z)dxdz \tag{7}$$

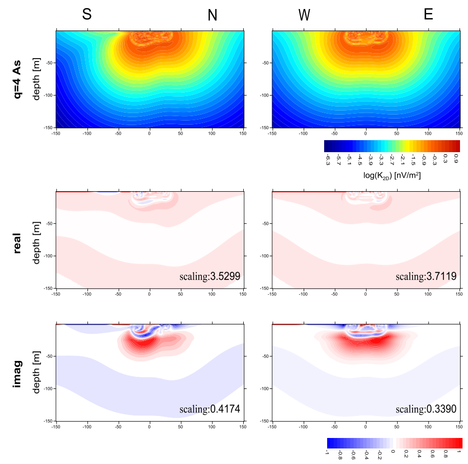

Figure 2: 2D kernels and their real and imaginary parts in South-North(left column) and West-East (right column) direction for a pulse moment of 4 As.The real and imaginary part plots are normalized to their maximum value, the respective scaling factors are displayed within each figure.The radius of circular loop is 24 m, the resistivity is 10Ω·m half-space and B_0 is the same as above.

METHOD AND RESULTS

Forward Modeling

Based on the above basic theory, figure 2 is the distribution of 2D spatial sensitivity not only including the signal amplitude but also the phase lag, i.e. the real and imaginary parts. Then we construct 4 synthetic models, perched water lens, dipping layered fracture aquifers, paleochannel and karst caverns (Figure 3). Each model is discretized into a set of rectangular cells of the same size (2×2m) and assuming a constant properties within each cell. The forward modeling is based on 7 equally spaced measure points (P1, P2, P3…P6, P7) along the profile. Among them, P4 is located at the middle of the profile (origin of the coordinates) and the interval between two measure points is 24 m. At each measure point, we utilize 21 pulse moments and the pulse moments are linear varied.

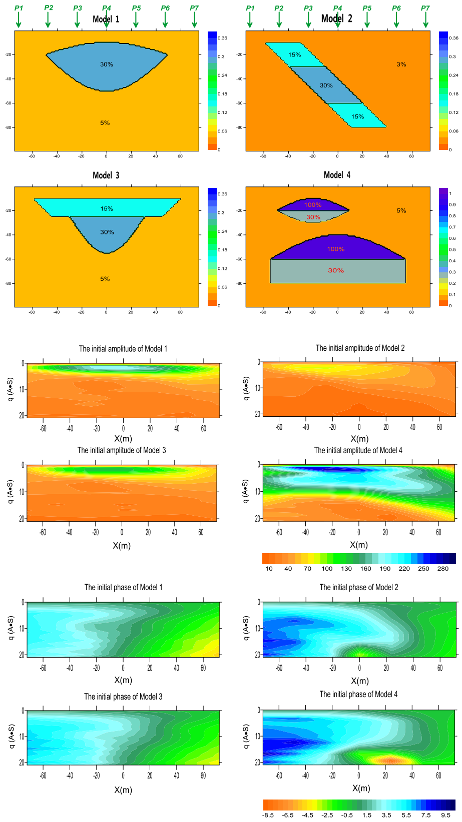

Figure 3: The four constructed synthtic models (above) and their initial amplitude and phase response (bottom). Modeling was performed for circular loops of 100m diameter on homogeneous half-space of 50Ω·m and B_0 is also 48000nT at 60◦ inclination, profile direction is N-S.

Inversion Method and Results

A commonly used solution is often referred to as the Tikhonov regularization solution. The function to be minimized consists of two terms taking both data fit and model smoothness into account

$$\left\|Gm^{est}-d^{obs}\right\|_2^2+\alpha^2\left\|Lm\right\|_2^2$$

For Marquardt method L=I (I: unit matrix), the model roughness is taken into account and referred to as the Tikhonov of zeroth order.

Figure 4 is the inversion results of the four synthetic models. The results indicate that the 2-D inversion scheme can recover the complex 2-D water-bearing models successfully, especially to reconstruct the accurate shallow water-bearing model. The resolution of 2-D SNMR tomography decrease with the increase of depth, and the recover models show reduced accuracy in the north direction of the profile than the south, particularly inversing only with real part data. This may be caused by the asymmetric intensity distribution of the transmitted oscillating magnetic field, which leads the tipping angle of the nuclear spins larger in the south than north, so there is a stronger response in the south. The upper high water content aquifer has a covering effect to the lower one, making the inversion result of the lower aquifer less certain. What's more, the imaginary part of SNMR data is illustrated to be more sensitive to the 2D structures than the real part and joint inversion offers the best resolution. We also add gaussian noise to the inversion process, the inversion results are impacted, but still can identify the basic characteristics of the aquifer.

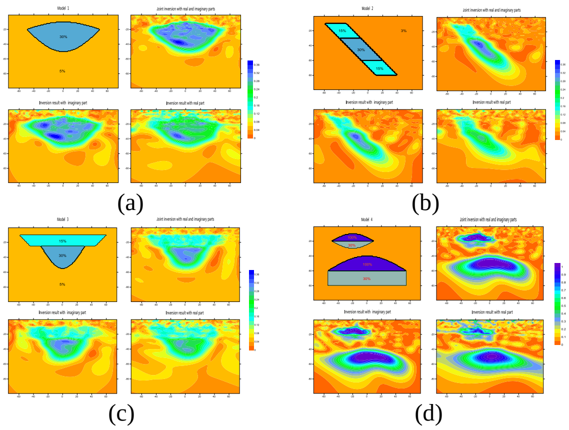

Figure 4:The inversion result of four models. For (a,b,c,d), upper left is the origin model, upper right is the joint inversion result with real and imaginary data, left lower is the inversion result with real data, right lower is the inversion result with imaginary data.

CONCLUSIONS

Through our research, we find that the Marquardt method can reconstruct the original complex 2D models correctly. The imaginary part of SNMR data is illustrated to be more sensitive to the 2D structures than the real part. Using imaginary data can provide useful and additional information in the inversion process. Joint inversion with complex data provides the most accurate results. The joint inversion not only stabilizes the inversion but also improves the inversion resolution. However, there are still some false water-bearing units within the inversion results. This paper is only a preliminary research of 2-D SNMR tomography, our method still exists some problems, so modifying the inversion method is our next research plan.

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REFERENCES

Hertrich, M., Braun, M., Günther, T., Green, A.G., and Yaramanci, U., 2007, Surface nuclear magnetic resonance tomography: Geoscience and Remote Sensing, IEEE Transactions, 45, 3752–3759.

Hertrich, M., Green, A.G., Braun, M., Yaramanci, U., 2009, High-resolution surface NMR tomography of shallow aquifers based on multioffset measurements: Geophysic, 74,G47-G59.

Legchenko, A.V., and Shushakov, O.A.,1998, Inversion of surface NMR data: Geophysics, 63,75-84.

Marian, H., 2005, Magnetic Resonance Sounding with Separated Transmitter and Receiver Loops for the Investigation of 2D Water Content Distributions: Ph.D. Thesis, Technische Universität Berlin.

Mohnke, O., and Yaramanci, U., 2002, Smooth and block inversion of surface NMR amplitudes and decay times using simulated annealing: Journal of Applied Geophysics, 50,163-177.

Weichman, P.B., Eugene, M.L., and Michael, H.R., 2000, Theory of Surface Nuclear Magnetic Resonance with Applications to Geophysical Imaging Problems: Physical Review E, 62, 1290-1312.

Yaramanci, U., Mueller-Petke, M., 2009,Improvements in inversion of magnetic resonance exploration-Water content, decay time, and resistivity: Journal of Earth Science, 20,592-605.